Inconsistency Management in Reactive Multi-Context Systems

Gerhard Brewka[1], Stefan Ellmauthaler[1], Ricardo Gonçalves[2], Matthias Knorr[2], João Leite[2], and Jörg Pührer[1]

[1] Institute of Computer Science, Leipzig University, Germany.
[2] NOVA LINCS & Departamento de Informática, Universidade NOVA de Lisboa, Portugal.

Abstract. We address the problem of global inconsistency in reactive multi-context systems (rMCSs), a framework for reactive reasoning in the presence of heterogeneous knowledge sources that can deal with continuous input streams. Their semantics is given in terms of equilibria streams. The occurrence of inconsistencies, where rMCSs fail to have an equilibria stream, can render the entire system useless. We discuss various methods for handling this problem, following different strategies such as repairing the rMCS, or even relaxing the notion of equilibria stream so that it can *go through* inconsistent states.

1 Introduction

The occurrence of inconsistencies within frameworks that aim at integrating knowledge from different sources cannot be neglected, even more so in dynamic settings where knowledge changes over time. In this paper, we deal with *reactive Multi-Context Systems (rMCSs)* [5,6,13] that allow for integrating heterogeneous knowledge bases with streams of incoming information and to use them for continuous online reasoning, reacting, and evolving the knowledge bases by internalizing relevant knowledge. There are many reasons why rMCSs may fail to have an equilibria stream. These include the absence of an acceptable belief set for one of its contexts given its current knowledge base, some occurring conflict between the operations in the heads of bridge rules, or simply because the input stream is such that the bridge rules prevent the existence of such an equilibria stream. We address the problem of inexistent equilibria streams, also known as *global inconsistency*. We begin by defining a notion of coherence associated with individual contexts which allows us to first establish sufficient conditions for the existence of equilibria streams, and then abstract away from problems due to specific incoherent contexts and focus on those problems essentially caused by the way the flow of information in rMCSs is organized through its bridge rules. We introduce the notion of a *repair*, which modifies an rMCS by changing its bridge rules at some particular point in time in order to obtain some equilibria stream, which we dub *repaired equilibria stream*. We establish sufficient conditions for the existence of repaired equilibria streams and briefly discuss different possible strategies to define such repairs. However, repaired equilibria streams may not always exist, because, for example, some particular context is incoherent. To deal with such situations, we relax the concept of equilibria stream and introduce the notion of *partial equilibria stream*, which essentially allows the non-existence of equilibria at some time points. It turns out that *partial equilibria streams* always exist thus solving the problem of global inconsistency for rMCSs.

2 Inconsistency Management

We assume that the reader is familiar with rMCSs and refer to [5] for a thorough discussion of their background and the notation used in the following.

In [8], the authors addressed the problem of global inconsistency in the context of *managed multi-context systems (mMCSs)* [4]. Just as we do here, they begin by establishing sufficient conditions for the existence of equilibria. Then, they define the notions of *diagnosis* and *explanation*, the former corresponding to bridge rules that need to be altered to restore consistency, and the latter corresponding to combinations of rules that cause inconsistency. These two notions turn out to be dual of each other, and somehow correspond to our notion of repair, the main difference being that, unlike in [8], we opt not to allow the (non-standard) strengthening of bridge-rule to restore consistency, and the fact that our repairs need to take into account the dynamic nature of rMCSs. We start by introducing two notions of global consistency differing only on whether we consider a particular input stream or all possible input streams.

Definition 1. *Let M be an rMCS,* KB *a configuration of knowledge bases for M, and $\mathcal{I}$ an input stream for M. Then, M is* consistent *with respect to* KB *and $\mathcal{I}$ if there exists an equilibria stream of M given* KB *and $\mathcal{I}$. M is* strongly consistent *with respect to* KB *if, for every input stream $\mathcal{I}$ for M, M is consistent with respect to* KB *and $\mathcal{I}$.*

Obviously, for a fixed configuration of knowledge bases, strong consistency implies consistency w.r.t. any input stream, but not vice-versa. Unfortunately, verifying strong consistency is in general highly complex since it requires checking all possible equilibria streams. Nevertheless, we can establish conditions that ensure that an rMCS M is strongly consistent with respect to a given configuration of knowledge bases KB, hence guaranteeing the existence of an equilibria stream independently of the input. It is based on two notions – *totally coherent contexts* and *acyclic rMCSs* – that together are sufficient to ensure (strong) consistency. Total coherence imposes that each knowledge base of a context always has at least one acceptable belief set.

Definition 2. *A context C_i is* totally coherent *if $\mathbf{acc}_i(kb) \neq \emptyset$, for every $kb \in KB_i$.*

The second notion describes cycles between contexts which may be a cause of inconsistency. Acyclic rMCSs are those whose bridge rules have no cycles.

Definition 3. *Given an rMCS $M = \langle\langle C_1, \dots, C_n\rangle, \mathsf{IL}, \mathsf{BR}\rangle$, $\triangleleft_M$ is the binary relation over contexts of M such that $(C_i, C_j) \in \triangleleft_M$ if there is a bridge rule $r \in BR_i$ and $j{:}b \in \mathrm{bd}(r)$ for some b. If $(C_i, C_j) \in \triangleleft_M$, also denoted by $C_i \triangleleft_M C_j$, we say that C_i depends on C_j in M. An rMCS M is* acyclic *if the transitive closure of $\triangleleft_M$ is irreflexive.*

These two conditions together are indeed sufficient to ensure strong consistency.

Proposition 1. *Let $M = \langle\langle C_1, \dots, C_n\rangle, \mathsf{IL}, \mathsf{BR}\rangle$ be an acyclic rMCS such that every C_i, $1 \leq i \leq n$, is totally coherent, and* KB *a configuration of knowledge bases for M. Then, M is strongly consistent with respect to* KB.

These conditions can be rather restrictive since there are many useful cyclic rMCSs which only under some particular configurations of knowledge bases and input streams may have no equilibria streams. To deal with these, and recover an equilibria stream, one possibility is to repair the rMCSs by locally, and selectively, eliminating some of its bridge rules. Towards introducing the notion of *repair*, given an rMCS $M = \langle\langle C_1, \ldots, C_n\rangle, \mathsf{IL}, \mathsf{BR}\rangle$, we denote by br_M the set of all bridge rules of M, i.e., $br_M = \bigcup_{1\leq i\leq n} BR_i$. Moreover, given a set $R \subseteq br_M$, denote by $M[R]$ the rMCS obtained from M by restricting the bridge rules to those not in R.

Definition 4 (Repair). *Let $M = \langle \mathsf{C}, \mathsf{IL}, \mathsf{BR}\rangle$ be an rMCS, KB a configuration of knowledge bases for M, and $\mathcal{I}$ an input stream for M until τ where $\tau \in \mathbb{N} \cup \{\infty\}$. Then, a repair for M given KB and $\mathcal{I}$ is a function $\mathcal{R} : [1..\tau] \to 2^{br_M}$ such that there exists a function $\mathcal{B} : [1..\tau] \to \mathsf{Bel}_M$ such that*

- *$\mathcal{B}^t$ is an equilibrium of $M[\mathcal{R}^t]$ given $\mathcal{KB}^t$ and $\mathcal{I}^t$, with $\mathcal{KB}^t$ inductively defined as*
  - $\mathcal{KB}^1 = \mathsf{KB}$
  - $\mathcal{KB}^{t+1} = \mathbf{upd}_{M[\mathcal{R}^t]}(\mathcal{KB}^t, \mathcal{I}^t, \mathcal{B}^t)$.

We refer to $\mathcal{B}$ as a repaired equilibria stream of M given KB, $\mathcal{I}$ and $\mathcal{R}$.

The notion of *repair* is quite general, and includes repairs that unnecessarily eliminate bridge rules, and even the *empty repair*, i.e., the repair $\mathcal{R}_\emptyset$ such that $\mathcal{R}_\emptyset^t = \emptyset$ for every t, whenever M already has an equilibria stream given KB and $\mathcal{I}$, ensuring that repaired equilibria streams properly extend equilibria streams.

Proposition 2. *Every equilibria stream of M given KB and $\mathcal{I}$ is a repaired equilibria stream of M given KB, $\mathcal{I}$ and the empty repair $\mathcal{R}_\emptyset$.*

It turns out that for rMCSs composed of totally coherent contexts, repaired equilibria streams always exist.

Proposition 3. *Let $M = \langle\langle C_1, \ldots, C_n\rangle, \mathsf{IL}, \mathsf{BR}\rangle$ be an rMCS such that each C_i, $i \in \{1, \ldots, n\}$, is totally coherent, KB a configuration of knowledge bases for M, and $\mathcal{I}$ an input stream for M until τ. Then, there exists $\mathcal{R} : [1..\tau] \to 2^{br_M}$ and $\mathcal{B} : [1..\tau] \to \mathsf{Bel}_M$ such that $\mathcal{B}$ is a repaired equilibria stream given KB, $\mathcal{I}$ and $\mathcal{R}$.*

Whenever repair operations are considered in the literature, e.g., in the context of databases [2], there is a special emphasis on seeking repairs that are somehow minimal, the rational being that we want to change things as little as possible to regain consistency. In the case of repairs of rMCS, we can establish an order relation between them, based on a comparison of the bridge rules to be deleted at each time point.

Definition 5. *Let $\mathcal{R}_a$ and $\mathcal{R}_b$ be two repairs for some rMCS M given a configuration of knowledge bases for M, KB and $\mathcal{I}$, an input stream for M until τ. We say that $\mathcal{R}_a \leq \mathcal{R}_b$ if $\mathcal{R}_a^i \subseteq \mathcal{R}_b^i$ for every $i \leq \tau$, and that $\mathcal{R}_a < \mathcal{R}_b$ if $\mathcal{R}_a \leq \mathcal{R}_b$ and $\mathcal{R}_a^i \subset \mathcal{R}_b^i$ for some $i \leq \tau$.*

This relation can be directly used to check whether a repair is minimal, and we can restrict ourselves to adopting minimal repairs. However, there may be good reasons to adopt non-minimal repairs, e.g., so that they can be determined *as we go*, or so that *deleted* bridge rules are not reinstated, etc. Even though investigating specific types of repairs falls outside the scope of this paper, we nevertheless discuss some possibilities.

Definition 6 (Types of Repairs). *Let* $\mathcal{R}$ *be a repair for some rMCS* M *given* KB *and* $\mathcal{I}$*. We say that* $\mathcal{R}$ *is a:*

Minimal Repair *if there is no repair* $\mathcal{R}_a$ *for* M *given* KB *and* $\mathcal{I}$ *such that* $\mathcal{R}_a < \mathcal{R}$*.*

Global Repair *if* $\mathcal{R}^i = \mathcal{R}^j$ *for every* $i, j \leq \tau$*.*

Minimal Global Repair *if* $\mathcal{R}$ *is global and there is no global repair* $\mathcal{R}_a$ *for* M *given* KB *and* $\mathcal{I}$ *such that* $\mathcal{R}_a < \mathcal{R}$*.*

Incremental Repair *if* $\mathcal{R}^i \subseteq \mathcal{R}^j$ *for every* $i \leq j \leq \tau$*.*

Minimally Incremental Repair *if* $\mathcal{R}$ *is incremental and there is no incremental repair* $\mathcal{R}_a$ *and* $j \leq \tau$ *such that* $\mathcal{R}_a^i \subset \mathcal{R}^i$ *for every* $i \leq j$*.*

Minimal repairs perhaps correspond to the ideal situation in that they never unnecessarily remove bridge rules. Sometimes, it may be the case that if a bridge rule is somehow involved in some inconsistency, it should not be used at any time point, leading to the notion of *global repair.* Given the set of all repairs, checking which are global is also obviously less complex than checking which are minimal. A further refinement – *minimal global repairs* – would be to only consider repairs that are minimal among the global ones, which would be much simpler to check than checking whether it is simply minimal. Note that a minimal global repair is not necessarily a minimal repair. One of the problems with these types of repairs is that we can only check whether they are of that type once we know the entire input stream $\mathcal{I}$. This was not the case with *plain* repairs, as defined in Def. 4, which could be checked *as we go*, i.e., we can determine what bridge rules to include in the repair at a particular time point by having access to the input stream $\mathcal{I}$ up to that time point only, important so that rMCSs can be used to effectively react to their environment. The last two types of repairs defined above allow for just that. *Incremental repairs* essentially impose that removed bridge rules cannot be reused in the future, while *minimally incremental repairs* further impose that only minimal sets of bridge rules can be added at each time point. Other types of repairs could be defined, e.g., based on a priority relation between bridge rules, or a distance measure between subsets of bridge rules. Repairs could also be extended to allow for the strengthening of bridge rules, besides their elimination, such as in [8] and [4].

Despite the existence of repaired equilibria streams for large classes of systems, two problems remain: first, computing a repair may be excessively complex, and second, there remain situations where no repaired equilibria stream exists, namely when the rMCS contains contexts that are not totally coherent. The second issue could be dealt with by ensuring that for each non-totally coherent context there would be some bridge rule with a management operation in its head that would always restore consistency of the context, and that such rule could always be *activated* through a repair. But this would require special care in the way the system is specified, and its analysis would require a very complex analysis of the entire system including the specific behavior of management functions. In practice, it would be quite hard – close to impossible in general – to ensure the existence of repaired equilibria streams, and we would still be faced with the first problem, that of the complexity of determining the repairs.

This can be addressed by relaxing the notion of equilibria stream so that it does not require an equilibrium at every time point. This way, if no equilibrium exists at some time point, the equilibria stream would be undefined at that point, but possibly defined again in subsequent time points, leading to the notion of *partial equilibria stream*.

Definition 7 (Partial Equilibria Stream). *Let* $M = \langle \mathsf{C}, \mathsf{IL}, \mathsf{BR} \rangle$ *be an rMCS,* KB *a configuration of knowledge bases for* M*, and* $\mathcal{I}$ *an input stream for* M *until* τ *where* $\tau \in \mathbb{N} \cup \{\infty\}$*. Then, a* partial equilibria stream of M given KB and $\mathcal{I}$ *is a partial function* $\mathcal{B} : [1..\tau] \nrightarrow \mathsf{Bel}_M$ *such that*

- $\mathcal{B}^t$ *is an equilibrium of* M *given* $\mathcal{KB}^t$ *and* $\mathcal{I}^t$*, with* $\mathcal{KB}^t$ *inductively defined as*
  - $\mathcal{KB}^1 = \mathsf{KB}$
  - $\mathcal{KB}^{t+1} = \begin{cases} \mathbf{upd}_M(\mathcal{KB}^t, \mathcal{I}^t, \mathcal{B}^t), & \text{if } \mathcal{B}^t \text{ is not undefined.} \\ \mathcal{KB}^t, & \text{otherwise.} \end{cases}$
- *or* $\mathcal{B}^t$ *is undefined.*

Partial equilibria streams generalize equilibria streams and do always exist.

Proposition 4. *Every equilibria stream of* M *given* KB *and* $\mathcal{I}$ *is a partial equilibria stream of* M *given* KB *and* $\mathcal{I}$*.*

Proposition 5. *Let* M *be an rMCS,* KB *a configuration of knowledge bases for* M*, and* $\mathcal{I}$ *an input stream for* M *until* τ*. Then, there exists* $\mathcal{B} : [1..\tau] \nrightarrow \mathsf{Bel}_M$ *such that* $\mathcal{B}$ *is a partial equilibria stream given* KB *and* $\mathcal{I}$*.*

Partial equilibria streams not only do allow us to deal with situations where equilibria do not exist at some time instants, they also open the ground to consider other kinds of situations where we do not wish to consider equilibria, for example because we were not able to compute them on time, or simply because we do not wish to process the input at every time point, e.g., whenever we just wish to sample the input with a lower frequency than it is generated. To restrict that partial equilibria streams only relax equilibria streams when necessary, we can further impose the following condition on Def. 7: $\mathcal{B}^t$ is undefined $\Rightarrow$ there is no equilibrium of M given $\mathcal{KB}^t$ and $\mathcal{I}^t$.

3 Conclusions

Following the efforts done in the combination of knowledge bases integration and knowledge dynamics [3,7,1,9,10,12,14,16], this paper addresses the problem of how inconsistencies can be managed within the framework of reactive Multi-Context Systems (rMCSs). The occurrence of inconsistencies within rMCSs cannot be neglected, especially as we deal with dynamic settings where knowledge changes over time. Even with the power of management operations that allow the specification of, e.g., belief revision operations, many reasons remain why rMCSs may fail to have an equilibria stream. Since the absence of equilibria at certain time points ultimately render the entire system useless, we addressed this problem first by showing sufficient conditions on the contexts and the bridge rules that ensure the existence of an equilibria stream. In the cases where these conditions are not met, we presented two possible solutions, one following an approach based on repairs and a second by relaxing the notion of equilibria stream to ensure that intermediate inconsistent states can be recovered. In future work, we would like to explore an alternative to deal with inconsistent states, following a paraconsistent approach, as proposed for hybrid knowledge bases in [11,15].

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