Humans Reason Skeptically

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Abstract The weak completion semantics is a novel cognitive theory. It is multi-valued, non-monotonic, knowledge rich, allows learning, can handle inconsistent background knowledge, and can be applied to model the average reasoner. Moreover, it uses abduction to explain observations, to satisfy integrity constraints, and to search for counterexamples. In all these applications, human reasoning tasks can only be adequately modeled within the weak completion semantics if skeptical abduction is applied, rather than credulous abduction. This will be illustrated in the context of the suppression task, disjunctive reasoning, and conditional reasoning.

1 Introduction

Two-valued classical logic is by now considered an *old* paradigm in the psychology of reasoning that has at least five fundamental problems, viz. it is knowledge-poor, background knowledge must be consistent, there can be no learning, it is mono-

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tonic, and it concentrates on individual reasoning [31]. Unfortunately, the cognitive science community has largely ignored modern developments within logic programming, logic-based knowledge-based systems, and machine learning, which provide solutions for the fundamental problems. An exception is the approach by [34], but unfortunately it contains a technical problem. This problem has been corrected by [17]. The solution was the basic step towards the *weak completion semantics* (WCS), a *new* paradigm which is multi-valued, non-monotonic, background knowledge need not to be consistent, and learning can be applied. It can also embody, for instance, abduction and counterfactual reasoning.

Given premises, general knowledge, and observations, *reasoning in the WCS* is modeled in six steps:

1. Reasoning towards a logic program $\mathscr{P}$ following [34].
2. Weakly completing the program to obtain $wc\mathscr{P}$.
3. Computing its least model $\mathscr{M}_{wc\mathscr{P}}$ under the three-valued [30] logic.
4. Reasoning with respect to $\mathscr{M}_{wc\mathscr{P}}$.
5. If known observations cannot be explained or given integrity constraints cannot be met applying skeptical abduction.
6. Searching for counterexamples.

On the other hand, logic programming and logic-based knowledge-based systems have largely ignored experimental findings in cognitive science. In this paper we consider experimental findings in human reasoning and show how the WCS can adequately model these findings in a comprehensive, computational, and formal theory. We will focus on abduction and will demonstrate that in all experiments humans seem to reason skeptically in line with the formal theory.

Among the established cognitive theories (see e.g. [26] for an overview) the mental model theory (see e.g. [19, 22, 27]) appears to be closest to the WCS. Comprehending premises, general knowledge, and perception, the mental model theory generates mental models and reasons with respect to these models. If no new knowledge can be inferred, then nothing new follows. Otherwise, the mental model theory validates the constructed models by searching for counterexamples. If no counterexamples can be found, then the previously generated putative conclusions are considered to be valid, otherwise they are considered to be invalid. In contrast to the mental model theory, the WCS is a formal theory and all steps are rigorously defined.

The paper is organized as follows. After an introductioon to the WCS in Section 2 we present three applications where adequate human reasoning can only be modeled by skeptical abduction: the suppression task in Section 3, human disjunctive reasoning in Section 4, and human conditional reasoning in Section 5. In the final Section 6 we discuss our results and relate them to the mental model theory.

2 The Weak Completion Semantics

We assume the reader to be familiar with logic and logic programming as presented in e.g. [11] and [28]. Let $\bot$, $\top$, and $\cup$ be truth constants denoting *truth*, *falsehood*, and *unknown*, respectively. A *(logic) program* is a finite set of clauses of the form $A \leftarrow Body$, where A is an atom and *Body* is either $\top$, or $\bot$, or a finite, non-empty set of literals. Clauses of the form $A \leftarrow \top$, $A \leftarrow \bot$, and $A \leftarrow L_1, \ldots, L_n$ are called *facts*, *assumptions*, and *rules*, respectively, where L_i, $1 \leq i \leq n$, are literals.

Throughout this paper, $\mathscr{P}$ will denote a program. An atom A is *defined* in $\mathscr{P}$ if and only if $\mathscr{P}$ contains at least a clause of the form $A \leftarrow Body$, otherwise A is *undefined*. We restrict our attention to propositional programs although the WCS extends to first-order programs [16, 8]. As an example consider the program

$$\mathscr{P}_0 = \{C \leftarrow A \wedge \neg ab,\ ab \leftarrow \bot\},$$

where A, C, and ab are atoms. C and ab are defined, whereas A is undefined. ab is an abnormality predicate which is assumed to be false. In the WCS, this program represents the conditional sentence *if A then C*. Thus, conditional sentences are not represented by implications but by *licenses for inferences* [34]. The abnormalities are initially assumed to false, but can later be used to represent–among others–enabling relationships (see e.g. [7]).

Consider the following transformation:

1. For all defined atoms A occurring in program $\mathscr{P}$, replace all clauses of the form $A \leftarrow Body_1$, $A \leftarrow Body_2$, $\ldots$ by $A \leftarrow Body_1 \vee Body_2 \vee \ldots$.
2. Replace all occurrences of $\leftarrow$ by $\leftrightarrow$.

The resulting set of equivalences is called the *weak completion* of $\mathscr{P}$ ($wc\,\mathscr{P}$). It differs from the completion defined in [3] in that undefined atoms are not mapped to false, but to unknown instead.

As shown by [17], each weakly completed program $wc\mathscr{P}$ admits a least model under the three-valued [30] logic. This model will be denoted by $\mathscr{M}_{wc\mathscr{P}}$. It can be computed as the least fixed point of a semantic operator introduced by [34]. Let $\mathscr{P}$ be a program and I a three-valued interpretation represented by the pair $\langle I^\top, I^\bot \rangle$, where $I^\top$ and $I^\bot$ are the sets of atoms mapped to true and false by I, respectively, and atoms which are not listed are mapped to unknown by I. Then, $\Phi_{\mathscr{P}}\, I = \langle J^\top, J^\bot \rangle$, [3] where

$$\begin{array}{rcl} J^\top &=& \{A \mid \text{there exists } A \leftarrow Body \in \mathscr{P} \text{ and } I\,Body = \top\}, \\ J^\bot &=& \{A \mid \text{there exists } A \leftarrow Body \in \mathscr{P} \text{ and} \\ && \quad \text{for all } A \leftarrow Body \in \mathscr{P} \text{ we find } I\,Body = \bot\}. \end{array}$$

Following [25], we consider an *abductive framework*

$$\langle \mathscr{P}, \mathscr{A}_{\mathscr{P}}, \mathscr{IC}, \models wcs \rangle,$$

[3] Whenever we apply a unary operator like $\Phi_{\mathscr{P}}$ to an argument like I, then we omit parenthesis and write $\Phi_{\mathscr{P}}\, I$ instead.

where $\mathscr{P}$ is a logic program,

$$\mathscr{A}_{\mathscr{P}} = \{A \leftarrow \top \mid A \text{ is undefined in } \mathscr{P}\} \cup \{A \leftarrow \bot \mid A \text{ is undefined in } \mathscr{P}\}$$

is the *set of abducibles*, $\mathscr{IC}$ is a finite set of *integrity constraints*, i.e. clauses of the form $U \leftarrow Body$ (*weak integrity constraint*) and $\bot \leftarrow Body$ (*strong integrity constraint*), and $\mathscr{M}_{wc\mathscr{P}} \models_{wcs} F$ if and only if $\mathscr{M}_{wc\mathscr{P}}$ maps the formula F to true. Let $\mathscr{O}$ be an *observation*, i.e., a finite set of literals. $\mathscr{O}$ is *explainable* in the abductive framework $\langle \mathscr{P}, \mathscr{A}_{\mathscr{P}}, \mathscr{IC}, \models_{wcs} \rangle$ if and only if there exists a non-empty $\mathscr{X} \subseteq \mathscr{A}_{\mathscr{P}}$ called *explanation* such that

1. $\mathscr{M}_{wc(\mathscr{P} \cup \mathscr{X})} \models_{wcs} L$ for all $L \in \mathscr{O}$ and
2. $\mathscr{M}_{wc(\mathscr{P} \cup \mathscr{X})}$ satisfies $\mathscr{IC}$, i.e., maps all clauses occurring in $\mathscr{IC}$ to true.

Sometimes explanations are assumed to be minimal, where an explanation $\mathscr{X}$ is *minimal* if there does not exist an explanation $\mathscr{X}'$ with $\mathscr{X}' \subset \mathscr{X}$. We note that if the set $\mathscr{O}$ of observations is empty, then the first condition is automatically satisfied, but not necessarily the second one. One should also observe that because each weakly completed program has a model under Łukasiewicz logic, $wc(\mathscr{P} \cup \mathscr{X})$ is satisfiable.

Formula F *follows credulously* from $\mathscr{P}$ and $\mathscr{O}$ iff there exists an explanation $\mathscr{X}$ for $\mathscr{O}$ such that $\mathscr{M}_{wc(\mathscr{P} \cup \mathscr{X})} \models_{wcs} F$. F *follows skeptically* from $\mathscr{P}$ and $\mathscr{O}$ iff $\mathscr{O}$ can be explained and for all explanations $\mathscr{X}$ for $\mathscr{O}$ we find $\mathscr{M}_{wc(\mathscr{P} \cup \mathscr{X})} \models_{wcs} F$. One should observe that if an observation $\mathscr{O}$ cannot be explained, then *nothing follows* credulously as well as skeptically. In the case of skeptical consequences the definition is an application of the so-called *Gricean implicature* [13]: humans normally do not quantify over things which do not exist, that is, at least one explanation $\mathscr{X}$ must exist.

3 The Suppression Task

The suppression task is a set of twelve experiments first carried out by [1] in the 1980s. The experiments showed that humans can suppress previously drawn conclusions when additional knowledge becomes available. This is independent of whether the previously drawn conclusions were valid or invalid with respect to classical two-valued logic.

The suppression task was the first experiment which was adequately modeled by the WCS by [7]. In the sequel we will discuss two of the twelve experiments to illustrate the WCS without and with abduction.

In one experiment, graduate students from Trinity College, University of Dublin, Republic of Ireland were given the conditional sentences

if she has an essay to write, then she will study late in the library,
if she has some textbooks to read, then she will study late in the library,

and the atomic sentence

she has an essay to write,

and were asked whether they are willing to draw one of the following conclusions:

she will study late in the library,
she will not study late in the library,
she may or may not study late in the library.

In the experiment, 96% of the participants selected the first answer.

In the WCS, the conditional sentences are modeled by the program $\mathscr{P}_1$ consisting of the clauses

$$\begin{aligned} \ell &\leftarrow e \wedge \neg ab_e, \\ \ell &\leftarrow t \wedge \neg ab_t, \\ ab_e &\leftarrow \bot, \\ ab_t &\leftarrow \bot, \end{aligned}$$

where ℓ, e, and t denote that *she is studying late in the library*, *she has an essay to write*, and *she has some textbooks to read*, respectively. ab_e as well as ab_t are abnormality predicates which are initially assumed to be false, thereby stipulating there are no known exceptions to each of the two rules. Adding the given atomic sentence to program $\mathscr{P}_1$ we obtain

$$\mathscr{P}_2 = \mathscr{P}_1 \cup \{e \leftarrow \top\}.$$

Weakly completing program $\mathscr{P}_2$ yields the set of equivalences

$$\begin{aligned} \ell &\leftrightarrow (e \wedge \neg ab_e) \vee (t \wedge \neg ab_t), \\ ab_e &\leftrightarrow \bot, \\ ab_t &\leftrightarrow \bot, \\ e &\leftrightarrow \top, \end{aligned}$$

whose least model under the three-valued Łukasiewicz logic is

$$\langle \{\ell, e\}, \{ab_e, ab_t\} \rangle,$$

i.e., ℓ and e are mapped to true, ab_e and ab_t are mapped to false, and t is mapped to unknown. Hence, we conclude that *she will be studying late in the library*.

In another experiment, graduate students were given the same set of conditional sentences, but the atomic sentence was replaced by

she is studying late in the library.

Then they were asked whether they were willing to draw one of the following conclusions:

she has an essay to write,
she does not have an essay to write,
she may or may not have an essay to write.

Because ℓ is defined in $\mathscr{P}_1$ and we do not want to change its definition, we cannot simply replace the fact $e \leftarrow \top$ in program $\mathscr{P}_s$ by the fact $\ell \leftarrow \top$. Rather, we consider *she is studying late in the library* as an observation, or query, in search of an explanation, i.e.,

$$\mathscr{O} = \{\ell\}.$$

The atoms e and t are undefined in program $\mathscr{P}_1$ and, thus, the set of their possible abducibles is

$$\mathscr{A}_{\mathscr{P}_1} = \{e \leftarrow \top,\ e \leftarrow \bot,\ t \leftarrow \top,\ t \leftarrow \bot\}.$$

There are two minimal explanations which can explain the observations, viz.

$$\{e \leftarrow \top\} \quad \text{and} \quad \{t \leftarrow \top\}.$$

Reasoning credulously we can respond to the above posed question *she has an essay to write*. Reasoning skeptically we can only respond *she may or may not have an essay to write*. Byrne reports that only 13% of the participants concluded *she has an essay to write*. Hence, the average participant cannot be adequately modeled by credulous abduction, but indeed by skeptical abduction.

If skeptical abduction is applied, then minimal explanations suffice. Non-minimal explanations like $\{e \leftarrow \top,\ e \leftarrow \bot\}$ or $\{e \leftarrow \top,\ t \leftarrow \top\}$ will not change the answer. On the other hand, computing non-minimal explanations requires additional resources and time.

4 Human Disjunctive Reasoning

In this section we will be concerned with sentences like *A or B, or both* called *disjunction*, and *A or B, but not both*, called *exclusive disjunction*. It is impossible to represent disjunctive sentences as programs because the head of each clause in a program must be an atom. However, it is possible to represent a disjunction by the strong integrity constraint

$$\bot \leftarrow \neg A \wedge \neg B$$

which according to Łukasiewicz logic is only satisfied if and only if A evaluates to true or B evaluates to true, or both. Likewise, an exclusive disjunction can be represented by the strong integrity constraints

$$\bot \leftarrow \neg A \wedge \neg B \quad \text{and} \quad \bot \leftarrow A \wedge B.$$

To satisfy both constraint either A must be true and B must be false or, vice versa A must be false and B must be true.

The following example is taken from [22]. Consider a disjunctive sentence like

Lisa is in Cambridge or Ben is in Dublin, or both.

We will consider the integrity constraint

$$\bot \leftarrow \neg c \wedge \neg d,$$

where c and d denote that *Lisa is in Cambridge* and *Ben is in Dublin*, respectively. This integrity constraint is not satisfied by models in which each of c and d are either mapped to false or unknown. At least one must be true. On the other hand, the negative atomic sentence

Lisa is not in Cambridge

can be represented by the program

$$\mathscr{P}_3 = \{c \leftarrow \bot\}.$$

Weakly completing this program yields

$$\{c \leftrightarrow \bot\},$$

whose least model under the three-valued Łukasiewicz logic is

$$\langle \emptyset, \{c\} \rangle,$$

i.e., c is mapped to false and d to unknown. Thus, this model does not satisfy the integrity constraint. The atom d is the only undefined atom occurring in program $\mathscr{P}_3$. Hence, we obtain

$$\mathscr{A}_{\mathscr{P}_3} = \{d \leftarrow \top,\ d \leftarrow \bot\}$$

as its set of abducibles. In the corresponding deductive framework, the empty observation

$$\mathscr{O} = \emptyset$$

cannot be explained because $\mathscr{I}\mathscr{C}$ is not satisfied, and we may ask whether there is a subset of $\mathscr{A}_{\mathscr{P}_3}$ such that $\mathscr{P}_3$ together with this subset explains $\mathscr{O}$. The minimal subset meeting this condition is $\{d \leftarrow \top\}$. Adding this subset to $\mathscr{P}_3$ we obtain

$$\mathscr{P}_4 = \mathscr{P}_3 \cup \{d \leftarrow \top\}.$$

Weakly completing $\mathscr{P}_4$ yields

$$\{c \leftrightarrow \bot,\ d \leftrightarrow \top\},$$

whose least model under the three-valued Łukasiewicz logic is

$$\langle \{d\}, \{c\} \rangle.$$

Hence, the WCS allows to conclude *Ben is in Dublin*.

The example can be modified to consider an exclusive disjunction instead of an inclusive disjunction [22]:

Linda is in Amsterdam or Cathy is in Majorca, but not both.
Cathy is in Majorca.

What follows? The two sentences can be modeled in exactly the same way as presented above except that the integrity constraints

$$\bot \leftarrow \neg a \wedge \neg m \quad \text{and} \quad \bot \leftarrow a \wedge m$$

are to be used, where a and m denote that *Linda is in Amsterdam* and *Cathy is in Majorca*, respectively. In this case, abduction will generate the least model

$$\langle \{m\}, \{a\} \rangle$$

and, hence, *Linda is not in Amsterdam* follows.

Consider the example:

Eva is in Rio or she is in Brazil, or both.
She is not in Brazil.

What follows? If modeled as the first example in this section, then WCS will conclude that *Eva is in Rio*. As pointed out by Phil N. Johnson-Laird and Ruth M. J. Byrne in [21] *no sensible person other than a logician is likely to draw this conclusion*. Well, a computer will also draw this conclusion if it was not informed that Rio is a city in Brazil. Hence, the background knowledge in this example should be

$$\mathscr{P}_5 = \{brazil \leftarrow rio\}.$$

In this program, *brazil* is defined and the negative atomic sentence *she is not in Brazil* should not be added to $\mathscr{P}_5$ as an assertion, but be treated as an observation which needs to be explained, viz.

$$\mathscr{O} = \{\neg brazil\}.$$

The first disjunctive sentence is represented by the integrity constraint

$$\bot \leftarrow \neg rio \wedge \neg brazil$$

and

$$\mathscr{A}_{\mathscr{P}_5} = \{rio \leftarrow \top,\ rio \leftarrow \bot\}.$$

It turns out that the observation $\neg brazil$ cannot be explained. The set

$$\{rio \leftarrow \bot\}$$

is discarded as an explanation: added to $\mathscr{P}_5$ and weakly completing the extended program we obtain

$$\{brazil \leftrightarrow rio,\ rio \leftrightarrow \bot\},$$

whose least model under Łukasiewicz logic is

$$\langle \emptyset, \{rio, brazil\} \rangle$$

and does not satisfy the integrity constraint. The set

$$\{rio \leftarrow \top\}$$

is also discarded as an explanation: added to $\mathscr{P}_5$ and weakly completing the extended program we obtain

$$\{brazil \leftrightarrow rio,\ rio \leftrightarrow \top\},$$

whose least model under Łukasiewicz logic is

$$\langle \{rio, brazil\}, \emptyset \rangle,$$

and does not entail the observation. The set $\mathscr{A}_{\mathscr{P}_5}$ is also discarded as an explanation: added to $\mathscr{P}_5$ and weakly completing the extended program we obtain

$$\{brazil \leftrightarrow rio,\ rio \leftrightarrow \top \vee \bot\},$$

whose least model under Łukasiewicz logic is again

$$\langle \{rio, brazil\}, \emptyset \rangle$$

and does not entail the observation. Hence, neither of the two exemplary statements follows.

A fourth example including disjunctions is taken from the German weekly newspaper *DIE ZEIT* which published the following puzzle on July 26, 2020 as part of a questionnaire testing the reader's intelligence:

Antonia is looking at Berta while Berta is looking at Cleopatra.
Antonia is wearing a red hat, Cleopatra is not wearing a hat,
and it is unknown whether Berta is wearing a red hat.
Is a person with a red hat looking at a person without a red hat?

Let a, b, and c denote *Antonia*, *Berta*, and *Cleopatra*, respectively, rX that X *is wearing a red hat*, $\ell(X,Y)$ that X *is looking at* Y, and the *goal* that *somebody with a red hat is looking at a person without a red hat*. The scenario can be formalized as a program $\mathscr{P}_6$ consisting of the following clauses:

$$\begin{aligned} \ell(a,b) &\leftarrow \top, \\ \ell(b,c) &\leftarrow \top, \\ r\,a &\leftarrow \top, \\ r\,c &\leftarrow \bot, \\ goal &\leftarrow \ell(X,Y) \wedge rX \wedge \neg rY. \end{aligned}$$

This is a first-order program. It can be turned into a propositional one by replacing the variables X and Y consistently by (all combination instances of) the constants a,

b, and c. The weak completion of this program admits the least model

$$\mathscr{M}_{wc\mathscr{P}_6} = \langle\{\ell(a,b), \ell(b,c), ra\}, \{rc\}\rangle$$

under Łukasiewicz logic. The fact that it is *unknown whether Berta is wearing a red hat* can be represented by the exclusive disjunction $rb \oplus \neg rb$ and, hence, by the integrity constraints

$$\bot \leftarrow rb \wedge \neg rb \qquad \text{and} \qquad \bot \leftarrow \neg rb \wedge \neg\neg rb.$$

Because conjunction is commutative and double negations can be removed, the two integrity constraints are semantically equivalent and it suffices to consider only the first one. Unfortunately, this integrity constraint is not satisfied by $\mathscr{M}_{wc\mathscr{P}_6}$ because $\mathscr{M}_{\mathscr{P}_6}$ maps rb to unknown. As rb is undefined in $\mathscr{P}_6$, the set $\mathscr{A}_{\mathscr{P}_6}$ of abducibles contains

$$rb \leftarrow \top \quad \text{and} \quad rb \leftarrow \bot.$$

The empty observation can be explained by either

$$\mathscr{X}_1 = \{rb \leftarrow \top\} \quad \text{or} \quad \mathscr{X}_2 = \{rb \leftarrow \bot\}.$$

We obtain

$$\begin{aligned} \mathscr{M}_{wc(\mathscr{P}_6 \cup \mathscr{X}_1)} &= \langle\{\ell(a,b), \ell(b,c), ra, rb, goal\}, \{rc\}\rangle, \\ \mathscr{M}_{wc(\mathscr{P}_6 \cup \mathscr{X}_2)} &= \langle\{\ell(a,b), \ell(b,c), ra, goal\}, \{rc, rb\}\rangle. \end{aligned}$$

Reasoning skeptically we conclude that *goal* is true: *there is a person with a red hat looking at a person without a red hat*. If rb holds, then *Berta* is the person in question; if rb does not hold, then it is *Antonia*. One should observe that reasoning skeptically we can neither conclude that *Berta is wearing a red hat* nor that *Berta is not wearing a red hat*.

More examples are discussed in [15], where it is also shown how disjunctive reasoning in the WCS is related to the elimination of disjunction in the calculus of natural deduction [12] and how the WCS avoids applications of the *falsum* rule.

5 Human Conditional Reasoning

In this section we consider four basic inference tasks. Each task constitutes two premises. The first premise is a single conditional *if A then C*, followed by the second, namely,

- A, that is the affirmation of the antecedent (AA) ,
- $\neg A$, that is the denial of the antecedent (DA) ,
- C, that is the affirmation of the consequent (AC) ,
- $\neg C$, that is the denial of the consequent (DC).

Reasoners are then asked to infer what follows from the premises. For example, what follows in each of the following reasoning problems?

1. *If it rains, then the roofs must be wet* and *it rains* (AA).
2. *If Paul rides a motorbike, then Paul must wear a helmet* and *Paul does not ride a motorbike* (DA).
3. *If the library is open, then Elisa is studying late in the library* and *Elisa is studying late in the library* (AC).
4. *If Nancy rides her motorbike, then Nancy goes to the mountains* and *Nancy does not go to the mountains* (DC).

The four cases are also covered in the suppression task (see Section 3) but in this section we will have a closer look at these reasoning problems, develop a classification of conditions, and apply this classification to adequately model the experimental data reported by [6] and [5]. Before doing so, however, we would like to discuss the question whether the second premise should be treated as a fact, an assumption, or an observation.

5.1 Facts, Assumptions, or Observations

Let us begin by reconsidering the program $\mathscr{P}_1$ in Section 3. Given the atomic sentence *she has an essay to write*, the same was added as a fact to $\mathscr{P}_1$ leading to $\mathscr{P}_2$, whereas the atomic sentence *she is studying late in the library* was represented as an observation. This is because unlike the former, in the latter case the atom ℓ was already defined by the rule $\ell \leftarrow e \wedge \neg ab_e$ representing the conditional *if she has an essay to write, then she will study late in the library*. Now the question that arises is, could we not represent the atomic sentence *she has an essay to write* as an observation instead? From a theoretical point of view, this is clearly possible as we will show in the following. In the program $\mathscr{P}_1$, e and t are undefined. Therefore its set of abducibles is

$$\mathscr{A}_{\mathscr{P}_1} = \{e \leftarrow \top,\ e \leftarrow \bot,\ t \leftarrow \top,\ t \leftarrow \bot\}.$$

There is only one minimal explanation for the observation e, viz.

$$\{e \leftarrow \top\}.$$

Adding this explanation to program $\mathscr{P}_1$ we obtain exactly the program $\mathscr{P}_2$ illustrated in Section 3. The same applies for a negated atomic sentence like, *she has no essay to write*.

Does it make a difference from a cognitive point of view whether the second premise in the AA and DA cases are treated as facts and assumptions, respectively, or as observations? We do not know as yet. Nevertheless, for the remainder of this subsection we assume that the second premise is considered as an observation.

5.2 The Semantics of Conditionals

The following example is taken from [29, 9] with minor modifications. Consider the conditional sentence

if it rains, then the roofs are wet and she takes her umbrella.

Its consequent is a conjunction of two atomic statements, viz. *the roofs are wet* and *she takes her umbrella*. Thus, it can be perceived as a combination of two conditionals with identical antecedents. Reasoning towards a logic program representing such a background knowledge may yield program $\mathscr{P}_7$ consisting of the following clauses:

$$\begin{aligned} wet\_roofs &\leftarrow rain \wedge \neg ab_r, \\ ab_r &\leftarrow \bot, \\ umbrella &\leftarrow rain \wedge \neg ab_u, \\ ab_u &\leftarrow \bot. \end{aligned}$$

We have assumed that the abnormalities with respect to roofs and umbrella are false as no information to the contrary is given. The least model of the weak completion of program $\mathscr{P}_7$ is

$$\langle \emptyset, \{ab_r, ab_u\} \rangle.$$

Because *rain* is the only undefined atom, the set of abducibles of program $\mathscr{P}_7$ is

$$\{rain \leftarrow \top,\ rain \leftarrow \bot\}.$$

The set of integrity constraints are empty.

Suppose, we consider the said conditional sentence as a first premise and supplement it, one by one, with the following atomic sentences acting as a second premise:

The roofs are not wet.
She does not take her umbrella.
The roofs are wet.
She takes her umbrella.

In the first two atomic statements, the consequent of the background knowledge is denied, whereas in the last two atomic statements, the consequent of the background knowledge is affirmed. What follows?

In the WCS each of the atomic statements are considered to be observations, and an attempt is made to explain them. For the first and the second atomic statement, the only minimal explanation is that *it has not rained*, whereas for the third and fourth atomic statement, the only minimal explanation is that *it has rained*. With this point of view, there is no apparent difference between *the roofs being wet* and *she is taking her umbrella*, in that they are both explained by the explanation that *it has rained*. However, in reality we are well aware that umbrellas can be used for various reasons like, for example, because *the sun is shining*. There is also more;

there is a general physical law that connects *rain* to the usual cause of wet roof tops whereas there is no physical law that connects *rain* to the cause of a person taking an umbrella. How can these differences be taken into account?

5.2.1 Obligational and Factual Conditionals

Consider the conditional

if it rains, then the roofs are wet.

Its consequent appears to be obligatory with respect to the antecedent. We cannot easily imagine a case, where the antecedent is true and the consequent is not. Following [2] such a conditional is called an *obligational conditional*. There are many examples of obligational conditionals:

If the roofs are not wet, then it did not rain.
If there is no light, then plants will not grow.
If an object is not supported, then it will fall to the ground.
If somebody's parents are elderly, then he/she should look after them [2].
If a German tourist wants to enter Russia, then he/she must hold a valid visa.
If somebody is riding a motorbike, then he/she must wear a helmet [2].
If a person is drinking beer, then he/she must be over 19 years of age [14].

The obligations have different sources. The last three are based on legal laws and are often called *deontic* conditionals. Others like the fourth are moral or social obligations. And the first three are causal or physical laws.

As pointed out by [2], for each obligational conditional there are two initial possibilities people think about, one of which is permitted and one of which is forbidden or unlikely (see Table 1). The first possibility is the conjunction of the antecedent and the consequent, which is permitted. The second possibility is the conjunction of the antecedent and the negation of the consequent, which is forbidden. Exceptions are possible but unlikely. For example, *it rains* and *the roof of a particular house does not get wet because it is under reconstruction and covered under a tarpaulin*. But it may be expected that such an exceptional case is usually mentioned explicitly.

It rains and the roofs are wet	permitted
It rains and the roofs are not wet	forbidden/unlikely

Table 1 The initial possibilities people think about for the obligational conditional *if it rains, then the roofs are wet*; adapted from [2].

Now consider the conditional

if it rains, then she takes her umbrella.

In this case, the possibility that *it rains* and *she does not take her umbrella* is not forbidden or unlikely. We can easily imagine a situation, where the antecedent is true and the consequent is not. For example, *she is driving to a supermarket with an underground parking lot*. Thus, the consequent is not obligatory with respect to the antecedent. Such a conditional is called a *factual conditional* by [2]. There are many examples of factual conditionals:

If she does not take her umbrella, then it will not rain.
If the sun is shining all day, then I will water my garden in the evening.
If the letter d is on one side of a card, then the number 3 is on the other side [35].
If Fred was in Paris, then Joe was in Lisbon [2].
If Nancy rides the motorbike, then she goes to the mountains [2].

In each of the examples we can easily come up with practical scenarios where the antecedent is true and the consequent is false. For example, even *if the sun is shining, one's neighbors may not water the garden in the evening because they are busy with something else*.

In general, humans may classify conditional sentences as obligational or factual. This is an informal and pragmatic classification. It depends on the background knowledge and experience of a person as well as on the context in which a conditional sentence is stated.

5.2.2 Necessary and Non-Necessary Antecedents

Let us consider the following conditional sentence

if it rains, then the roofs are wet.

Here, the antecedent appears to be necessary in order for the consequent to be true. That is, the consequent *the roofs are wet* cannot be true unless the antecedent *it rains* is true. At least, we cannot easily imagine a situation where *the roofs are wet* and *it has not rained*. It is possible that *the roofs are wet because the fire brigade watered it in an exercise.* But as mentioned earlier, this is quite an exceptional case and we can expect that such an exception would have been mentioned in the background knowledge.

Formally, the antecedent A of a conditional *if* A *then* C is said to be *necessary* if and only if its consequent C cannot be true unless the antecedent is true. Other examples are the antecedents of the conditional sentences

if there is gas in the gas tank, then the (combustion) engine will start

and

if the switch is toggled, then the light will be turned on.

Each of these antecedents are necessary with respect to the consequent. The *engine will not start* unless *there is gas in the gas tank*. Likewise, *the light will not be turned on* unless *the switch is toggled*. As so often, there may be exceptional cases. *If the starter of a car with a regular (non-diesel) engine is broken then you may tow the car and release the clutch in the second gear to have the engine started*. But this should have been mentioned.

On the other hand, the antecedent of conditional

if it rains, then she takes her umbrella

does not appear to be necessary. There are many different reasons for *taking an umbrella* like, for example, that *the sun is shining*. In other words, it is easily imaginable that the lady takes her umbrella regardless of whether it rains or not. The antecedent is therefore *non-necessary*. Other examples are the antecedents of the conditionals

if I want to meet friends, then I will go to my favorite pub

and

if I want to have a little something to eat, then I will go to my favorite pub.

In both cases, the antecedents are non-necessary for *going to my favorite pub*. It may well be that *I will go to my favorite pub* even if *I do not want to meet friends* or *I do not want to have a little something to eat.* I may just want to drink a beer before going to sleep. In other words, in both cases it is imaginable that the consequent is true regardless of the truth of the antecedent.

In general, humans may classify antecedents as necessary or non-necessary. The classification is informal and pragmatic. It depends on the background knowledge and experience of a person as well as on the context in which the condition is stated. This implies that a person's classification may change if these change.

5.2.3 The Influence of Pragmatics

Summarizing all that has been discussed in the previous sections, humans may generally recognize conditional sentences as obligational or factual and antecedents as necessary or non-necessary. This leads to an informal and pragmatic classification of four kinds: obligational conditional with necessary antecedent or non-necessary antecedent and factual conditional with necessary antecedent or non-necessary antecedent. For an abstract conditional *if* A *then* C, without an everyday context, the classification of the conditional into any of the aforementioned kinds would be quite straightforward. The classification of everyday conditionals, however, often depends on pragmatics: the context, the background knowledge and experience of a person. For example, the conditional sentence *if it is cloudy, then it is raining*

discussed in [27] may be classified as an obligational conditional with necessary antecedent by people living in Java, whereas it may be classified as a factual conditional by people living in Central Europe. In another example [21], the authors conducted an experiment, where they categorized the proposition

if it's heated, then this butter will melt

as a bi-conditional. In particular they considered

if butter is not heated, it will not melt.

This corresponds to a necessary antecedent in our setting. Whereas some of their subjects also gave it the same classification, many considered it possible that even

if butter is not heated (explicitly), it may still melt.

This implies that they considered the antecedent to be non-necessary.

5.2.4 Representing the Semantics of Conditionals

Obligational and factual conditionals are represented using programs as before. Thus, a conditional $if\, A\, then\, C$, where A and C are ground atoms, is represented by the program $\mathscr{P}_0$ consisting of the clauses

$$\begin{aligned} C &\leftarrow A \wedge \neg ab, \\ ab &\leftarrow \bot. \end{aligned}$$

However, the semantics of conditionals is taken into consideration by extending the set of abducibles for a given program $\mathscr{P}$:

$$\mathscr{A}^e_\mathscr{P} = \mathscr{A}_\mathscr{P} \cup \mathscr{A}^{nn}_\mathscr{P} \cup \mathscr{A}^f_\mathscr{P},$$

where

$$\begin{aligned} \mathscr{A}^{nn}_\mathscr{P} &= \{C \leftarrow \top \mid && C \text{ is head of a rule in } \mathscr{P} \text{ representing} \\ & && \text{a conditional with non-necessary antecedent}\}, \\ \mathscr{A}^f_\mathscr{P} &= \{ab \leftarrow \top \mid && ab \text{ occurs in the body of a rule in } \mathscr{P} \\ & && \text{representing a factual conditional}\}. \end{aligned}$$

The set $\mathscr{A}^{nn}_\mathscr{P}$ contains facts for the consequents of conditionals with non-necessary antecedent. If an antecedent of a conditional is non-necessary then there may be other (unknown) reasons for establishing the consequent of the conditional. The set $\mathscr{A}^f_\mathscr{P}$ contains facts for the abnormalities occurring in the representation of factual conditionals. The antecedent of a factual conditional may be true, yet the consequent of the conditional may still not hold. Adding a fact for the abnormality occurring in the body of the representation of a factual conditional will force this abnormality to become true and its negation to become false. Hence, the body of the clause containing the abnormality predicate will be false. Table 2 illustrates the new facts in the set of abducibles.

$C \leftarrow A \wedge \neg ab$	A non-necessary	A necessary
Factual conditional	$ab \leftarrow \top$, $C \leftarrow \top$	$ab \leftarrow \top$
Obligational conditional	$C \leftarrow \top$	

Table 2 The additional facts in the set of abducibles for a rule of the form $C \leftarrow A \wedge \neg ab$ representing a conditional *if A then C*, where A and C are ground atoms and ab is an abnormality predicate.

5.3 An Experiment

In [6] and [5] an experiment concerning conditional reasoning is described, where 56 logically naive participants were tested on an online website (Prolific, `prolific.co`). The participants were restricted to Central Europe and Great Britain to have a similar background knowledge about weather etc. It was also assumed that the participants had not received any education in logic beyond high school training. The participants were first presented with a story followed by two assertions. The first was a conditional premise, and the second was an atomic premise. In some cases, this atomic premise was negated. Finally, for each problem they had to answer the question "What follows?". Both parts were presented simultaneously. The participants responded by clicking one of the answer options. They could take as much time as they needed. Participants acted as their own controls. Each participant solved all four inference types (AA, DA, AC, DC) for each of the 12 conditionals listed in the Appendix, thereby carrying out 48 inference tasks in total. For each task they could select one of three responses: *nothing follows (nf)*, the fact that had not been presented in the second premise, and the negation of this fact. For example in the case of a DA inference task, the first assertion was of the form *if A then C*, the second assertion was $\neg A$, and they could choose among answers C, $\neg C$ or nf. The classification of the 12 conditionals into the four aforementioned kinds,

- obligational conditional with necessary antecedent,
- obligational conditional with non-necessary antecedent,
- factual conditional with necessary antecedent,
- factual conditional with non-necessary antecedent,

was done by the authors beforehand, and was not mentioned to the participants.

To further clarify what the tasks looked like, please consider the following excerpt taken from the original experiment:

Peter has a lawn in front of his house. He is keen to make sure that the grass on the lawn does not dry out, so whenever it has been dry for multiple days, he turns on the sprinkler to water the lawn.

Along with this context the conditional sentence

if it rains, then the lawn is wet

and the negated atomic proposition

it does not rain

were provided. (The reader might recognize that this is a DA inference task). Following this, the participants were given three choices of answers: *the lawn is wet*, *the lawn is not wet* and *nothing follows*. The time taken by the participants to respond with their choice was also recorded in milliseconds.

5.3.1 Affirmation of the Antecedent

The results for the affirmation of the antecedent are shown in Table 3. A substantial number of the participants, that is 95% (640 out of 672) answered C. There was no significant deviation with respect to the classification of the conditional sentences. It appears that the participants performed a *modus ponens* inference. Reconsider the program $\mathscr{P}_0$ from 5 and recall that

$$\mathscr{A}_{\mathscr{P}_0} = \{A \leftarrow \top,\ A \leftarrow \bot\}.$$

The observation A can be explained by

$$\{A \leftarrow \top\}.$$

Weakly completing the extended program

$$\mathscr{P}_0 \cup \{A \leftarrow \top\}$$

and computing its least model yields

$$\langle \{A, C\}, \{ab\} \rangle. \tag{1}$$

We conclude C, as done by 95% of the participants in the experiment.

At this point, while the application of modus ponens seems to be a standard in the AA inference task, we would also like to redirect the attention of the reader to the notion of counterexamples. In order to do so we start by revisiting the concept of factual conditional sentences, as discussed in this paper. In such a case, given A, a situation where C may not hold is imaginable and may serve as a counterexample to the putative conclusion C. Within the WCS framework, such counterexamples in case of factual conditionals may be accounted for by extending the set of abducibles to include $ab \leftarrow \top$. As an example, let us consider the conditional sentence

Conditional/Classification	C	$\neg C$	*nf*	Sum	*Mdn C*	*Mdn nf*
Obligational Conditional	*black*!10323	9	*black*!104	336	3516	4183
Factual Conditional	*black*!10317	7	*black*!1012	336	3640	6575
Necessary Antecedent	320	8	8	336	3546	6926
Non-necessary Antecedent	320	8	8	336	3588	4934
Total	640	16	16	672	3570	5925

Table 3 The results for AA inferences given a conditional sentence *if A then C* and an atomic fact A. *Mdn C* and *Mdn nf* show the median response time in milliseconds for C and *nf*, respectively.

if the sun is shining all day, then I will water my garden in the evening.

It is a factual conditional. We can easily imagine a situation, where the sun has been shining all day, but (contrary to what the conditional suggests) we do not water our garden in the evening. Let us consider a sunny, winter day in Germany when temperatures are below zero degrees Celsius. Most people would probably not want to water their gardens in the biting cold afterwards. In other words, it is fathomable that such conditionals may, at least theoretically, lead some individuals to consider counterexamples to the consequent of the conditional. In the current example, the program $\mathscr{P}_0$ 5 can again be used to represent the conditional premise. The observation that the sun is shining all day may be explained by

$$\{A \leftarrow \top\},$$

which leads to the least model

$$\langle\{A, C\}, \{ab\}\rangle. \tag{2}$$

But it can also be explained by

$$\{A \leftarrow \top,\ ab \leftarrow \top\}.$$

In the latter case, the weak completion of the extended program

$$\mathscr{P}_0 \cup \{A \leftarrow \top,\ ab \leftarrow \top\}$$

admits the least model

$$\langle\{A, ab\}, \emptyset\rangle. \tag{3}$$

Comparing the models (2) and (3) we cannot skeptically conclude that C holds. But, the data from the current experiment (in Table 3) suggests that apparently very few humans do this. There is a decrease in C answers from 323 to 317, and an increase in *nothing follows* answers, from 4 to 12 (see grey cells) when switching from an obligational to a factual conditional. This is however, not statistically significant. It appears that most humans are so well-adapted to modus ponens inferences that they do not search for counterexamples during an AA inference. In fact psychologists often describe the affirmation of the antecedent inference task *as* modus ponens [10, 21]. That said, it must also be conceded that none of the conditional sentences from the experiment explicitly used words like *possibly*, *might* or *may*, which as discussed in [21] may indicate to the reasoner the possibility that even if A is true, C may not be true. In principle, replacing the above conditional with *if the sun is shining all day, then I will possibly water my garden in the evening*, should result in a modeling similar to the one illustrated above. The conjecture remains to be verified however, and creates scope for future experiments. [4]

5.3.2 Denial of the Antecedent

The results of the aforementioned experiment with regard to the DA are shown in Table 4. Everyday contexts for the DA inference task elicited a high response rate of about 78% (525 out of 672) for $\neg C$, but in case of *nf* the rate varied from 8% (28 out of 336) up to 30% (101 out of 336). The number of participants answering C seems irrelevant.

Conditional/Classification	C	$\neg C$	*nf*	Sum	*Mdn* $\neg C$	*Mdn nf*
Obligational Conditional	12	254	70	336	3583	6613
Factual Conditional	6	271	59	336	3518	6221
Necessary Antecedent	8	*black*!10300	*black*!1028	336	3474	5808
Non-Necessary Antecedent	10	*black*!10225	*black*!10101	336	3646	6700
Total	18	525	129	672	3558	6450

Table 4 The results for DA inferences given a conditional sentence *if A then C* and a negated atomic fact $\neg A$. *Mdn* $\neg C$ and *Mdn nf* show the median response time in milliseconds for $\neg C$ and *nf*, respectively. It may be observed that when the antecedent is non-necessary, *nf* is answered significantly more often, whereas the number for $\neg C$ answers decreases (see grey cells).

The reader might first observe that as per the experiment data *nothing follows* was answered much more often in case of conditional sentences with non-necessary antecedents than in the case of conditional sentences with necessary ones (30% vs. 8%, Wilcoxon signed rank, $W = 0$, $p < .001$). More importantly, as soon as the classification of the antecedent changed from necessary to non-necessary, the number of $\neg C$ responses decreased to 225 and *nf* simultaneously increased to (a significant) 101.

Although it is currently impossible to fully ascertain the exact reasoning steps that go on inside the mind of a human reasoner, the behavior of the data at hand

[4] The example nicely illustrates the role played by an abnormality predicate. Among other uses, it is also used to encode enabling relationships like that the temperatures must be above zero degrees Celsius to water a garden.

suggests two reasoning patterns. Firstly, most humans generally conclude $\neg C$ given a DA task and stop the reasoning process at that point, regardless of the classification of the conditional or nature of the antecedent. Secondly, there are significantly many who depending upon the necessity of the antecedent move on to the second stage and consider counterexamples to the putative conclusion drawn in the first. Hence, the *necessity* of the antecedent of a conditional seems to be a relevant feature which plays an influencing role in the DA task. Aside from this, the fact that the median response time for a *nf* response during a DA inference is longer than a $\neg C$ response (see Table 4), supplements the possibility that humans may be considering counterexamples and thus deal with more than one model when responding *nothing follows*. All that has been said so far may be best explained using the following examples from the experiment.

Given the conditional sentence

if plants get water, then they will grow

and the atomic sentence

the plants do not get water.

The following program represents the conditional premise

$$\mathscr{P}_8 = \{g \leftarrow w \wedge \neg ab_g,\ ab_g \leftarrow \bot\},$$

where g and w denote that *the plants will grow* and *the plants get water*, respectively, and ab_g is an abnormality predicate. The conditional sentence is classified as an obligational conditional with necessary antecedent. Hence, its set of abducibles is

$$\mathscr{A}_{\mathscr{P}_8} = \{w \leftarrow \top,\ w \leftarrow \bot\} = \mathscr{A}^e_{\mathscr{P}_8}.$$

The second premise, $\neg w$, is taken to be an observation that needs to be explained. The set of integrity constraints is empty. The only minimal explanation is

$$\{w \leftarrow \bot\}.$$

Adding this explanation to $\mathscr{P}_8$, weakly completing the extended program, and computing its least model we obtain

$$\langle \emptyset,\ \{g, ab_g, w\}\rangle.$$

Here nothing is true, and g, ab_g and w are all false. The observation *the plants do not get water* is thus explained, and, we also conclude that *the plants will not grow*.

Most humans seem to draw this conclusion and stop reasoning at this point in general, which is indicated by the majority $\neg C$ responses in the DA task. However, in order to fully account for the DA experiment results, we need to be able to account for the increase in the *nothing follows* responses when the antecedent of a conditional sentence changes from necessary to non-necessary. This leads us to the question, if it is plausible that humans look for counterexamples to their previously drawn conclusion when they deem the antecedent of conditional sentence to be non-necessary. The example illustrated above is unfortunately not a good fit to comprehend this question because for many humans the given antecedent may be necessary. Thus, we turn to the following example from the experiment.

Given the conditional premise

> *if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age*

and the negated atomic premise

> *Maria is not drinking alcoholic beverages in a pub.*

In the WCS the conditional premise is represented by

$$\mathscr{P}_9 = \{o \leftarrow a \wedge \neg ab_o,\ ab_o \leftarrow \bot\},$$

where o and a denote that *Maria is over 19 years old* and *she is drinking alcoholic beverages*, respectively, and ab_o is an abnormality predicate which is initially assumed to be false. Because the conditional sentence is classified as an obligation with non-necessary antecedent, we obtain

$$\mathscr{A}_{\mathscr{P}_9} = \{a \leftarrow \top,\ a \leftarrow \bot\} \quad \text{and} \quad \mathscr{A}^e_{\mathscr{P}_9} = \mathscr{A}_{\mathscr{P}_9} \cup \{o \leftarrow \top\}.$$

The negated atomic premise, $\neg a$, is considered as an observation that needs to be explained. The set of integrity constraints is empty. Considering $\mathscr{A}_{\mathscr{P}_9}$ the observation is explained by the minimal explanation

$$\{a \leftarrow \bot\}. \tag{4}$$

Adding this explanation to $\mathscr{P}_9$, weakly completing the extended program, and computing its least model we obtain

$$\langle \emptyset,\ \{o, ab_o, a\}\rangle \tag{5}$$

and conclude, that *Maria is younger than 19 years of age*. However, if a reasoner is searching for counterexamples, then he may discover a second explanation, viz.

$$\{a \leftarrow \bot,\ o \leftarrow \top\}. \tag{6}$$

Although Maria is older than 19 years of age she is not drinking alcoholic beverages. Adding this explanation to $\mathscr{P}_9$, weakly completing the extended program, and computing its least model we obtain

$$\langle \{o\},\ \{ab_o, a\}\rangle. \tag{7}$$

Comparing the least models (5) and (7) and reasoning skeptically, we conclude *nothing follows*. Comparing the explanations (4) and (6) we find

$$\{a \leftarrow \bot\} \subset \{a \leftarrow \bot,\ o \leftarrow \top\}.$$

The second explanation is a superset of the first one.

5.3.3 Affirmation of the Consequent

The results of the aforementioned experiment with regard to the AC reasoning task are shown in Table 5. Given the premises *if A then C* and C, that is an AC task with an everyday context, a majority of the participants answered A. However, like in the DA, the *necessity* of the antecedent seems to be a relevant feature of the conditional in the case of AC. This observation is suggested by the fact that the number of A responses decreased from 266 to 199 when the classification of the antecedent changed from necessary to non-necessary. More importantly, the number of *nothing follows* responses simultaneously increased to 129. This suggests two reasoning patterns, influenced by the necessity of the antecedent. Moreover, the fact that the median response time for a *nothing follows* response during an AC inference is longer than a A response (see Table 5), supplements the possibility that humans may be considering counterexamples and thus deal with more than one model when responding *nothing follows*.

Conditional/Classification	A	$\neg A$	*nf*	Sum	*MdnA*	*Mdnnf*
Obligation Conditional	231	6	99	336	3888	6044
Factual Conditional	234	8	94	336	3769	5650
Necessary Antecedent	*black*!10266	6	*black*!1064	336	3735	5450
Non-Necessary Antecedent	*black*!10199	8	*black*!10129	336	3906	6039
Total	465	14	193	672	3826	5802

Table 5 The results for AC inferences given a conditional sentence *if A then C* and an atomic fact C. *MdnA* and *Mdnnf* show the median response time in milliseconds for A and *nf*, respectively. It may be observed that when the antecedent is non-necessary, *nf* is answered significantly more often, whereas A is answered less often (see grey cells).

We begin by revisiting the conditional sentence

if plants get water, then they will grow

and the given atomic sentence

the plants grow.

The following program represents the conditional premise

$$\mathscr{P}_{10} = \{g \leftarrow w \wedge \neg ab_g,\ ab_g \leftarrow \bot\},$$

where g and w denote that *the plants will grow* and *the plants get water*, respectively, and ab_g is an abnormality predicate. The second premise, g, is taken to be an observation that needs to be explained. The conditional sentence is classified as an obligational conditional with *necessary* antecedent. Hence, its set of abducibles is

$$\mathscr{A}_{\mathscr{P}_{10}} = \{w \leftarrow \top,\ w \leftarrow \bot\} = \mathscr{A}^e_{\mathscr{P}_{10}}.$$

The set of integrity constraints is empty. The only minimal explanation for the observation g is

$$\{w \leftarrow \top\}.$$

Adding this explanation to $\mathscr{P}_{10}$, weakly completing the extended program, and computing its least model we obtain

$$\langle \{g,\ w\},\ \{ab_g\} \rangle.$$

Here w and g are true, whereas ab_g is false. We thus conclude that *the plants get water*.

Most humans seem to draw this conclusion and stop reasoning at this point. This is the general case, as indicated by a majority of A responses in the AC task. However, in order to fully account for the experiment results of the AC, we need to be able to account for the increase in the *nothing follows* responses when the antecedent of a conditional sentence changes from necessary to non-necessary. We stipulate that it is when humans are able to come up with models counter-exemplary to their previously generated model that they reason skeptically among them and choose to respond *nothing follows*. Similar to what was observed in the DA task, in case of the AC it seems plausible that this said search for counterexamples happens when humans deem the antecedent of a conditional sentence to be non-necessary. The present example is unfortunately not a good fit to illustrate this as many people would probably consider its antecedent to be necessary and thus fail to have counter-exemplary possibilities even if deliberately searching for them. Therefore we turn to the following example from the experiment, which may be more appropriate for our purpose.

Given the conditional premise

> *if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age*

and the atomic premise

> *Maria is over 19 years of age*.

In the WCS the conditional premise is represented by

$$\mathscr{P}_{11} = \{o \leftarrow a \wedge \neg ab_o,\ ab_o \leftarrow \bot\},$$

where o and a denote that *Maria is over 19 years old* and *she is drinking alcoholic beverages*, respectively, and ab_o is an abnormality predicate which is initially assumed to be false. The atomic premise, o, is considered as an observation that needs to be explained. Because the conditional sentence is classified as an obligation with *non-necessary* antecedent, and a is undefined in $\mathscr{P}_{11}$, we obtain,

$$\mathscr{A}_{\mathscr{P}_{11}} = \{a \leftarrow \top,\ a \leftarrow \bot\} \quad \text{and} \quad \mathscr{A}^{e}_{\mathscr{P}_{11}} = \mathscr{A}_{\mathscr{P}_{11}} \cup \{o \leftarrow \top\}.$$

Considering $\mathscr{A}_{\mathscr{P}_{11}}$, the observation is explained by the minimal explanation

$$\{a \leftarrow \top\} \tag{8}$$

Adding this explanation to $\mathscr{P}_{11}$, weakly completing the extended program, and computing its least model we obtain

$$\langle \{o, a\},\ \{ab_o\} \rangle \tag{9}$$

and conclude, that *Maria is drinking alcoholic beverages in a pub*. However, if a reasoner is searching for counterexamples, then she may discover a second explanation to the observation, viz.

$$\{o \leftarrow \top\}. \tag{10}$$

Here o being true signifies the possibility that *Maria might still be over 19 irrespective of whether she is drinking alcohol in a pub or not*. Adding such an explanation to $\mathscr{P}_{11}$, weakly completing the extended program, and computing its least model we obtain

$$\langle \{o\},\ \{ab_o\} \rangle. \tag{11}$$

Comparing the least models (9) where a is true and (11) where a is unknown, and reasoning skeptically, we conclude *nothing follows*. In other words, those who find counterexamples which suggest that although o holds a need not necessarily hold, conclude *nothing follows*. One should observe that the explanations (8) and (10) are independent in that neither is a subset nor a superset of the other.

5.3.4 Denial of the Consequent

The results of the aforementioned experiment with regard to the DC reasoning task are shown in Table 6. Given the premises *if A then C* and $\neg C$, that is a DC task with an everyday context, a majority of the participants answered $\neg A$. However, the type of the conditional sentence, whether obligational or factual, seems to be a relevant feature for those who may be reasoning skeptically. This observation is suggested by the fact that the number of $\neg A$ responses decreased from 277 to 234 when the classification of the conditional sentence changed from obligational to factual. More importantly, the number of *nothing follows* responses simultaneously increased to 96. This suggests two reasoning patterns, influenced by the type of the conditional. Moreover, the fact that the median response time for a *nothing follows* response during a DC inference is longer than a $\neg A$ response (see Table 6), supplements the possibility that humans may be considering counterexamples and thus deal with more than one model when responding *nothing follows*.

Conditional/Classification	A	$\neg A$	nf	Sum	$Mdn\,\neg A$	$Mdn\,nf$
Obligation Conditional	7	*black*!10277	*black*!1052	336	4053	4790
Factual Conditional	6	*black*!10234	*black*!1096	336	4459	4345
Necessary Antecedent	7	267	62	336	4096	4758
Non-Necessary Antecedent	6	244	86	336	4325	4555
Total	13	511	148	672	4311	5162

Table 6 The results for DC inferences given a conditional sentence *if A then C* and a negated atomic fact $\neg C$. $Mdn\,\neg A$ and $Mdn\,nf$ show the median response time in milliseconds for $\neg A$ and *nf*, respectively. It may be observed that when the conditional is factual, *nf* is answered more often, whereas there are less $\neg A$ answers (see grey cells).

Let us consider the following conditional sentence from the experiment,

if Maria is drinking alcoholic beverages in a pub, then Maria must be over 19 years of age

and the atomic premise

Maria is not over 19 years of age.

In the WCS the conditional premise is represented by

$$\mathscr{P}_{12} = \{o \leftarrow a \wedge \neg ab_o,\ ab_o \leftarrow \bot\},$$

where o and a denote that *Maria is over 19 years old* and *she is drinking alcoholic beverages*, respectively, and ab_o is an abnormality predicate which is initially assumed to be false. The negated atomic premise, $\neg o$, is considered as an observation that needs to be explained. As the conditional sentence is classified as an *obligational* conditional with non-necessary antecedent, and a is undefined in $\mathscr{P}_{12}$, we obtain,

$$\mathscr{A}_{\mathscr{P}_{12}} = \{a \leftarrow \top,\ a \leftarrow \bot\} \quad \text{and} \quad \mathscr{A}^e_{\mathscr{P}_{12}} = \mathscr{A}_{\mathscr{P}_{12}} \cup \{o \leftarrow \top\}.$$

The set of integrity constraints is empty. Considering $\mathscr{A}_{\mathscr{P}_{12}}$ the observation is explained by the minimal explanation

$$\{a \leftarrow \bot\}. \tag{12}$$

Adding this explanation to $\mathscr{P}_{12}$, weakly completing the extended program, and computing its least model we obtain

$$\langle \emptyset, \{o, a, ab_o\} \rangle \tag{13}$$

Thus we conclude, that *Maria is not drinking alcoholic beverages in a pub*.

Most humans seem to draw this conclusion and stop reasoning at this point. This is the general case, as indicated by a majority of $\neg A$ responses in the DC task. However, we would also like to be able to account for the increase in the *nothing follows* responses when the type of a conditional sentence changes from obligational to factual. Like in the case of the DA and the AC, we stipulate that it is when humans are able to come up with models counter-exemplary to their previously generated model, that they reason skeptically among them and choose to respond *nothing follows*. However, in case of the DC it is the (obligational or factual) type of the conditional which plausibly influences this said search for counterexamples. The present example is likely to be deemed an obligational conditional for many people who may thus fail to find counterexamples during deliberation. Therefore it is not a good fit to illustrate the suggested point. We turn to the following example from the experiment in order to better illustrate skeptical reasoning in the case of DC inference tasks.

Given the conditional sentence

if Ron scores a goal, then he is happy

and the given atomic sentence

Ron is not happy.

The following program represents the conditional premise

$$\mathscr{P}_{13} = \{h \leftarrow g \wedge \neg ab_h,\ ab_h \leftarrow \bot\},$$

where g and h denote *Ron scores a goal* and *Ron is happy*, respectively, whereas ab_h is an abnormality predicate. Here, $\neg h$ is considered as an observation that needs to be explained. The conditional sentence is classified as a *factual* conditional with non-necessary antecedent, and g is undefined in $\mathscr{P}_{13}$, therefore,

$$\mathscr{A}_{\mathscr{P}_{13}} = \{g \leftarrow \top,\ g \leftarrow \bot\} \quad \text{and} \quad \mathscr{A}^e_{\mathscr{P}_{13}} = \mathscr{A}_{\mathscr{P}_{13}} \cup \{h \leftarrow \top,\ ab_h \leftarrow \top\}.$$

The set of integrity constraints is empty. The observation $\neg h$ is explained by the minimal explanation

$$\{g \leftarrow \bot\}. \tag{14}$$

Adding this explanation to $\mathscr{P}_{13}$, weakly completing the extended program, and computing its least model we obtain

$$\langle \emptyset, \{h, g, ab_h\} \rangle \tag{15}$$

and conclude, that *Ron does not score a goal*. This is where most reasoners seem to halt their reasoning. However, there may be some individuals who recognize the conditional sentence as factual, meaning, they recognize that $\neg h$ need not just be caused or explained by $\neg g$. More precisely, they recognize that h may be false, whereas g is not. Analogously, they search for counterexamples to the putative conclusion $\neg g$, which can also explain $\neg h$. Within the WCS framework,

$$\{ab_h \leftarrow \top\} \tag{16}$$

can be used as an another explanation for $\neg h$. Here ab_h being true indicates that *Ron may have other reasons to be unhappy*. Adding this abducible to $\mathscr{P}_{13}$ and weakly completing the resulting extended program leads to the least model

$$\langle\{ab_h\},\ \{h\}\rangle. \tag{17}$$

As g is false in the first model (15) whereas unknown in the second, (17), skeptical reasoning is applied which leads to the conclusion, *nothing follows*. The overall point of importance is that, aside from the falsity of g it is also possible to find other reasons which can cause h to be false, and this leads to the consideration of more than one model, which may lead humans to reason skeptically. Comparing the explanations (14) and (16) we find that they are independent.

6 Discussion

Abduction is a form of plausible reasoning, in a general sense, because humans need it to come up with likely consistent explanations for observations. They also need to discover consistent abducible actions to achieve their goals. Additionally, they need to come up with reasoned explanations, or justifications, for their ethical choices of actions or for their ethical opinions, possibly having to adopt abductions about facts as yet unknown, in situations where full information is missing. Likewise, humans often need justifications for abducing preferences having consequences (say, chosing coffee over tea in some particular context), or exercising preferences amongst available abducibles, ruling out ones in favor of others, once they realize the situational models afforded by such choices. By consequences we refer to the consequences that would follow from adopted abductions, or the abductions themselves. Moreover, humans may need to abduce consequential expectations about what might happen independently of them, as they are making their choices.

In this chapter we have considered three different forms of human reasoning: the suppression or previously drawn inferences, human disjunctive reasoning, and human conditional reasoning. In each form, skeptical abduction had to be applied in order to adequately model the reasoning process of humans. The list of example could be extended. For example, the WCS models human syllogistic reasoning better if skeptical abduction is applied [26, 32]

The WCS, while being a formal theory, is one that by its wide available flexibility, as our many examples and experiments convincingly purport to show, exhibits the makings of a rigorous plausible cognitive model, having the ability of being further extended and refined, as experience so solicits it, whilst retaining its significant prior achievements, its logical grounding, its mathematical properties, and its inspiration in ongoing artificial intelligence knowledge representation and reasoning.

Speaking of cognitive theories, the discussion remains incomplete without a mention of the Mental Model Theory. The notion of mental models has gained momentum ever since the 1950s after psychologist Kenneth Craik's intriguing work, "The Nature of Explanation" [4] on internal models of the world, which aid humans in their thought process and decision-making. His arguments about the human brain's decision-making relying on internal models of the world, led to the inception of the (modern) Mental Model Theory (MMT) [18], which has been subject to evolution since then. Broadly speaking, the MMT is an informal, cognitive theory based on the central idea that much of human reasoning depends on mental models that the brain constructs based on the perception or a description of the real world. Till date, it has discussed various areas of human reasoning in terms of mental models; for example, [24, 19, 20, 23] to name a few. In particular, according to the MMT, humans reason based on mental models which in turn represent various possibilities of the states of the world. This notion of mental models is relatively informal in comparison to formal logic. But, if the human mind does indeed depend on mental models for its reasoning, it begs the question whether there is a no way to reconcile such a fact with a formal and computational framework? The fact that classical, two-valued logic is insufficient to explain or model human reasoning has been purported by many logicians. However, there is more to formal logic than the classical two-valued logic which is at times subject to criticism as also discussed by [33]. The WCS is such a novel, formal, cognitive theory which relies upon three-valued semantics based on Łukasiewicz logic in its goal to comprehensively explain and model the much-discussed areas of human reasoning.

Both the MMT and the WCS are non-monotonic, as is human reasoning. Furthermore, the WCS recognizes the fact that Johnson-Laird and Byrne have discussed at length in their work on the meanings of conditionals [21], which is, how a reasoner comprehends a conditional sentence depends upon the pragmatics of the utterance of the sentence, the reasoner's own background knowledge and experience etc. This is termed as *modulation* by the authors. This also implies that, the set of possibilities that characterize a conditional sentence in the mind of an individual may be different from the set of possibilities that characterize the same sentence in the mind of another. In terms of logic, the models being constructed in the mind of one reasoner may differ from those being constructed in the mind of another, depending upon their understanding of the world.

Both the MMT and the WCS agree on the notion of counterexamples. As suggested by [19], given a set of premises, if one is beginning to form a conclusion, one should believe or adopt the conclusion only if they are able to find no counterexamples that are strong enough to refute it. The data gathered from the experiment discussed in this paper, with the inclusion of the median response times, suggests that the reasoners responding *nothing follows* may be doing so. The notion of counterexamples or in particular counter-exemplary models, as modeled in the WCS, also brings to light the relevance of the nature of the antecedent in the case of DA and the AC inference tasks, and the relevance of the factual nature of a conditional, while reasoning in the DC inference task. The AA inference task shows a ceiling effect, the reasons of which motivate further research. However, the possibility that a reasoner who comprehends a conditional sentence as factual during an AA task, may respond *nothing follows*, is kept open in the present discussion.

Our analysis of the MMT and the WCS with regard to the four conditional inference tasks, following [21], have shown that in case of *deliberation* (as so called by the MMT) the overall predictions of the theory coincide with that of the WCS. The WCS identifies this deliberative phase with the search for counterexamples and attributes the *nothing follows* responses to skeptical reasoning resulting from the same. The results for what the MMT refers to as *intuition*, which may be identified with the first stage of the reasoning process *before* the search for counterexamples in the WCS, differs for the two theories. This is further purported by an analysis of the experimental data and an attempt to fit the predictions (for intuition) made by the MMT to it, which raises a few questions somewhat in spirit of the observations made by [10]. This however, calls for a different kind of discussion which is beyond the scope of the current.

In comparison, things look more favorable between the two theories when it comes to the suppression task. The task broadly illustrates how the introduction of a second conditional sentence such as *if B then C*, alongside the initial conditional *if A then C* can suppress the modus ponens inference of C given A, or even the DA inference of $\neg C$ and the AC inference of A. In [1] Ruth Byrne maintains that the suppression depends on how the individual in question integrates the second conditional into the mental model arising from the first and is thereby influenced by the context of the situation, the individual's background knowledge etc. Broadly speaking, if B is treated as an additional requirement (for C) alongside A, the mental model of B will be combined with that of A. Hence satisfying only A is no longer sufficient to conclude C and thus modus ponens is suppressed. While modeling the task, the WCS in its part takes this additional requirement B to be an *enabler* and models it using the abnormality predicate. This abnormality predicate is taken into consideration before concluding C, and signifies the room for skepticism in the mind of the human reasoner. The conclusion is no longer C, but rather nothing follows. Comprehending B as an alternative antecedent on the other hand leads individuals to suppress DA and AC inferences. This is because the mental model of B is no longer joined with A but rather considered independently to it. This is treated in a similar fashion within the WCS framework where the weak completion of the two conditional leads to a disjunction between the two, signifying the antecedent A or B non-necessary in itself, with respect C.

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Appendix: The Conditionals of the Experiment

Obligational Conditionals with Necessary Antecedent	
(1)	*If it rains, then the roofs must be wet.*
(2)	*If water in the cooking pot is heated over* 99°*C, then the water starts boiling.*
(3)	*If the wind is strong enough, then the sand is blowing over the dunes.*
Obligational Conditionals with Non-Necessary Antecedent	
(4)	*If Paul rides a motorbike, then Paul must wear a helmet.*
(5)	*If Maria is drinking alcoholic beverages in a pub,* *then Maria must be over 19 years of age.*
(6)	*If it rains, then the lawn must be wet.*
Factual Conditionals with Necessary Antecedent	
(7)	*If the library is open, then Sabrina is studying late in the library.*
(8)	*If the plants get water, then they will grow.*
(9)	*If my car's start button is pushed, then the engine will start running.*
Factual Conditionals with Non-Necessary Antecedent	
(10)	*If Nancy rides her motorbike, then Nancy goes to the mountains.*
(11)	*If Lisa plays on the beach, then Lisa will get sunburned.*
(12)	*If Ron scores a goal, then Ron is happy.*