FAKULTÄT
FÜR !NFORMATIK
Faculty of Informatics

Database and Artificial Intelligence Group

Advanced SAT Techniques for Abstract Argumentation

Johannes Peter Wallner   Georg Weissenbacher   Stefan Woltran

CLIMA XIV - September 16, 2013

Funded by

Motivation

Argumentation is a central part of AI with many applications

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E-Democracy tools

Parmenides

SUPPORTING DEMOCRATIC DECISION-MAKING

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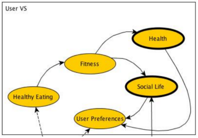

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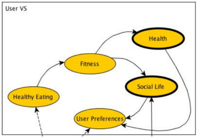

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Conference Series

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dbai

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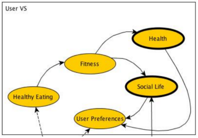

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Further applications

- Decision support systems [Amgoud and Prade, 2009]
- Medical Sciences [Hunter and Williams, 2012]
- Legal reasoning [Bench-Capon and Prakken, 2010]

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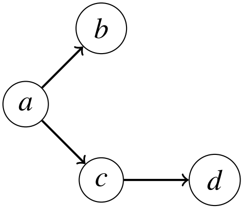

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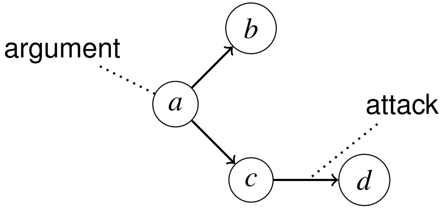

Formal Models of Argumentation

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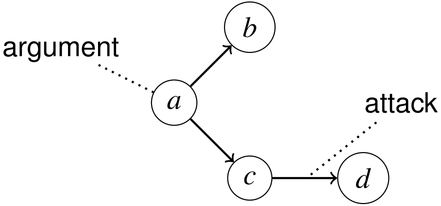

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- Criteria for acceptance usually based on admissibility and maximality

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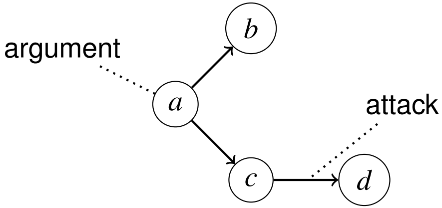


- Main reasoning task: select arguments which are jointly acceptable
- Criteria for acceptance usually based on admissibility and maximality
- Almost all tasks are intractable, often even “beyond” NP
- This calls for efficient solving algorithms

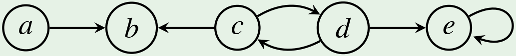
Argumentation Frameworks

Argumentation Framework [Dung, 1995]

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks")

Example



Argumentation Semantics

Semantics specify acceptable subsets $S \subseteq A$ of an AF $F = (A, R)$. Many semantics are available for different purposes

- Preferred
- Stable
- Semi-Stable
- Ideal
- Eager
- ...

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Basic Properties

Semantics are usually based on two important properties

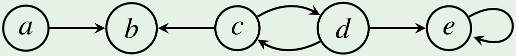
1. Conflict-freeness
2. Admissibility

Basic Properties

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

Example



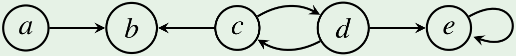
$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$

Admissible Sets

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example



$adm(F) = \{\{a, c\}, \{a, d\},$ ~~$\{b, d\}$~~$, \{a\},$ ~~$\{b\}$~~$, \{c\}, \{d\}, \emptyset\}$

Range

Given an AF $F = (A, R)$. The range of a set $S \subseteq A$ is
$S_R^+ = S \cup \{a \mid (b, a) \in R \text{ and } b \in S\}$

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Semi-Stable Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of F, if

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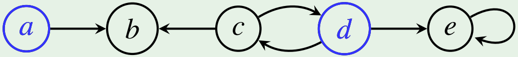

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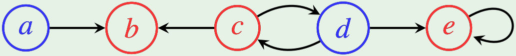

$semi(F) = \{\{a, d\}\}$ and $\{a, d\}_R^+ = \{a, b, c, d, e\}$

Reasoning Tasks and Complexity

Reasoning Tasks

Given an AF $F = (A, R)$ and $a \in A$ and semantics σ.

- a is credulously accepted w.r.t. σ if $a \in \bigcup \sigma(F)$
- a is skeptically accepted w.r.t. σ if $a \in \bigcap \sigma(F)$

σ	Cred_σ	Skept_σ
adm	NP-c	trivial
semi	Σ_2^p-c	Π_2^p-c

Boolean Satisfiability

- Satisfiability of a boolean formula is a famous NP-complete problem
- Many problems can be reduced to SAT

$$(a \lor b \lor \neg c) \land (\neg a \lor \neg d)$$

satisfied with assignment (model)

$$a \mapsto T, b \mapsto T, c \mapsto T, d \mapsto F$$

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- Result of about 15 years engineering and research are very efficient SAT solvers
- Biennial SAT competition furthers development

SAT and AFs

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- Suitable for several reasoning tasks on AFs

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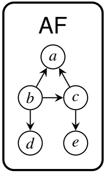
Instance



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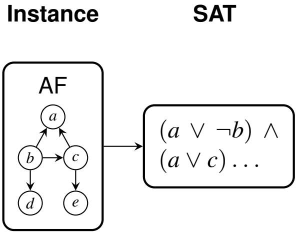

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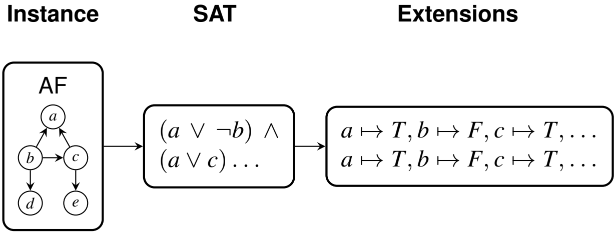

Complex Problems on AFs

- Many semantics are based on the concept of admissibility and a notion of maximality
- Admissibility + maximality usually corresponds to problem beyond NP $\Rightarrow$ cannot be compactly encoded in a single formula

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Need different formalism

- More expressive language (e.g. QBF, disjunctive ASP)
- iterative SAT (e.g. as in CEGARTIX system for abstract argumentation or a recent system presented at TAFA'13)

Simplified pseudocode, every check is done using a SAT solver

1. initialize $\mathcal{V} = \emptyset$
2. While there exists an admissible set S, with $S_R^+ \not\subseteq V_R^+$ for any $V \in \mathcal{V}$
   a. While there exists an admissible set T with $S_R^+ \subset T_R^+$

      $S \leftarrow T$

   b. S is a semi-stable extension; add it to $\mathcal{V}$
3. return $\mathcal{V}$

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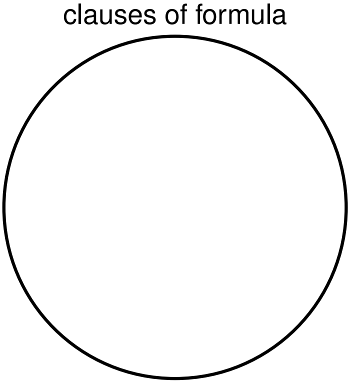
NP-search engine is delegated to SAT-solver, but

- Not as declarative as e.g. QBF
- requires engineering effort

Using the SAT technique of minimal correction sets we are in between CEGARTIX and QBF approaches

Hard and Soft Constraints

- Normally a SAT solver has to satisfy all clauses of a given formula



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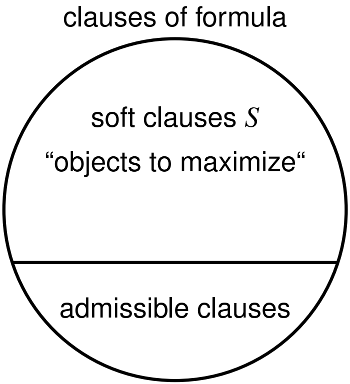

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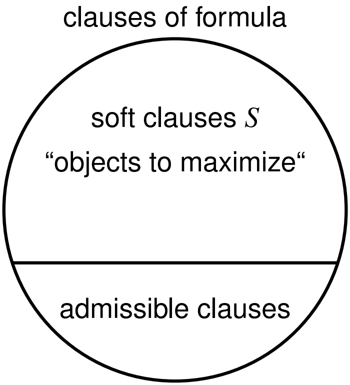

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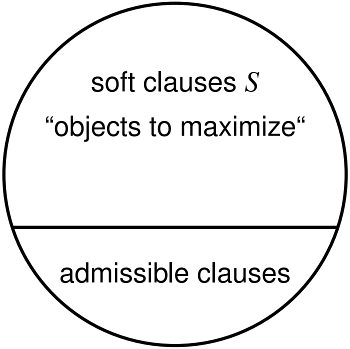
clauses of formula



$\wedge\ atMost(0, S)$

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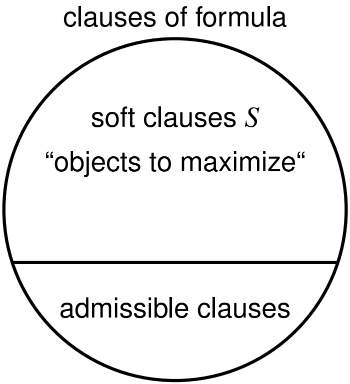

$\wedge \; atMost(0, S)$

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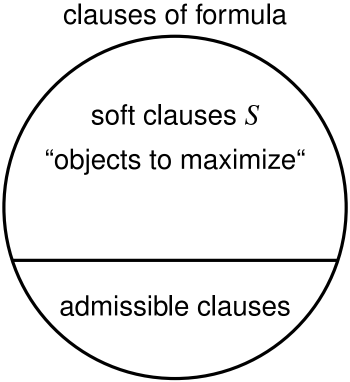

$\wedge\ atMost(0, S)$

$\wedge\ atMost(1, S)$

$\wedge\ atMost(2, S) \wedge$ ”block previous“

$\vdots$

Encoding the Range

Range

Given an AF $F = (A, R)$. The range of a set $S \subseteq A$ is
$S_R^+ = S \cup \{a \mid (b, a) \in R \text{ and } b \in S\}$

$$in\_range_{a,R} = (a \vee \bigvee_{(b,a)\in R} b)$$

Encoding Semi-stable Semantics

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$$adm_{A,R} \wedge \bigwedge_{a \in A} in\_range_{a,R}$$

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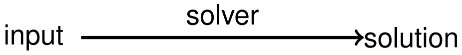
$$adm_{A,R} \wedge \bigwedge_{a \in A} in\_range_{a,R}$$

- Range encoding as soft clauses

Iterative SAT Optimizations



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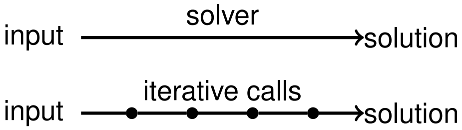

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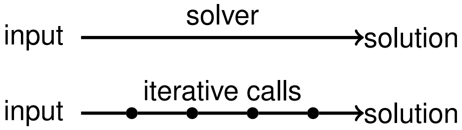
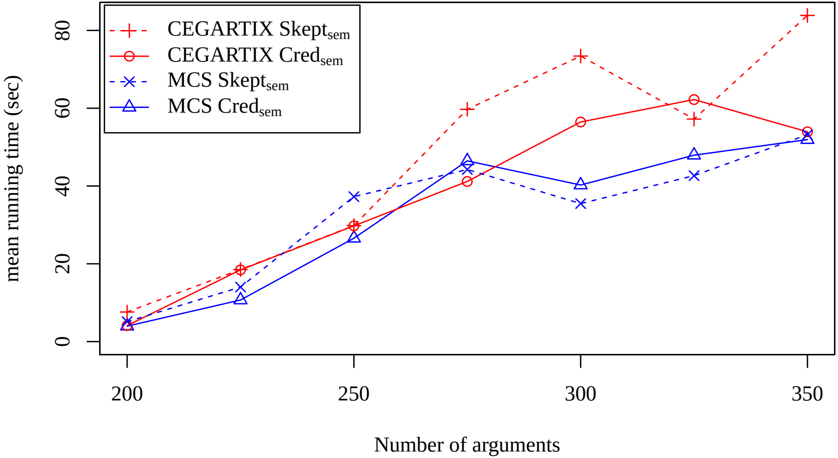


- Potential benefits of iterative SAT:
  - if the input satisfies certain properties, then the number of calls is low
  - Possibility to ”guide“ the algorithm at each call
- Preliminary performance tests are promising

Experiment Setup

- Implementation of
  - Semi-stable
- in a C++ prototype using
  - CAMUS v1.0.5
- Reasoning tasks:
  - Check for credulous/skeptical acceptance
- Performance tests
  - vs CEGARTIX v0.1a
- Instances
  - Randomly generated AFs


CEGARTIX Skept$_{sem}$
CEGARTIX Cred$_{sem}$
MCS Skept$_{sem}$
MCS Cred$_{sem}$
mean running time (sec)
0
20
40
60
80
200
250
300
350
Number of arguments

Conclusion

Many SAT techniques available; in our system we utilize

- Minimal correction sets for semi-stable and eager semantics and
- Backbones for ideal semantics (using JediSAT v0.2 beta)
- solving enumeration and cred/skept reasoning

SAT techniques appear to be very useful to solve various problems in argumentation (and potentially in AI)

The presented system is available at

`www.dbai.tuwien.ac.at/proj/argumentation/sat-based/`

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