Adding Closed World Assumptions to Well Founded Semantics

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Abstract

Given a program P we specify an enlargement of its Well Founded Model which gives meaning to the adding of Closed World Assumptions. We do so by proposing the desirable principles of a Closed World Assumption (CWA), and proceed to formally define and apply them to Well Founded Semantics (WFS), in order to obtain a WFS added with CWA, the O-semantics. After an introduction and motivating examples, there follow the presentation of the concepts required to formalize the model structure, the properties it enjoys, and the criteria and procedures which allow the precise characterization of the preferred unique maximal model that gives the intended meaning to the O-Semantics of a program, the O-Model. Some properties are also exhibited that permit a more expedite obtention of the models. Several detailed examples are introduced throughout to illustrate the concepts and their application. Comparison is made with other work, and in the conclusions the novelty of the approach is brought out.

1 Introduction and Motivation

Well Founded Semantics [Van Gelder *et al.*, 1980] has been proposed as a suitable semantics for general logic programs. Its Extended Stable Models (XSM) [Przymusinska and Przymusinski, 1990, Przymusinski, 1990] version has been explored as a framework for formalizing a variety of forms of non-monotonic reasoning [Pereira *et al.*, 1991d, Pereira *et al.*, 1991e] and generalized to deal with contradiction removal and counterfactuals [Pereira *et al.*, 1991a, Pereira *et al.*, 1991b, Pereira *et al.*, 1991c]. The increasing rôle of logic programming extensions as an encompassing framework for these and other AI topics is expounded at length in [Kakas and Mancarella, 1991b], where they argue, and we concur, that WFS is by design overly careful in deciding about the falsity of some atoms, leaving them undefined, and that a suitable form of CWA can be used to safely and undisputably assume false some of the atoms absent from the well founded model of a program. Consider the program P, adapted from [Kakas and Mancarella, 1991a]: $\{a \leftarrow \sim a;\ c \leftarrow \sim a\}$, where $WFM(P) = \{\}$. We argue that the intended meaning of the program may be $\{\sim c\}$, since $\sim a$ may not be true in any model of P, by the first rule, and so, the second rule cannot contradict the assigned meaning. Another way to understand this is that one may safely assume $\sim c$ using a form of CWA on c, since $\sim a$ may not be consistently assumed.

However, when relying on the absence of present evidence about some atom A, we do not always want to assume that $\sim A$ holds, since there may exist consistent assumptions allowing to conclude A. Roughly, we want to define the notion of concluding for the truth of a negative literal $\sim A$ just in case there is no hard nor hypothetical evidence to the contrary, i.e. no consistent set of negative assumptions such that $\sim A$ is untenable.

Consider $P = \{a \leftarrow \sim b;\ b \leftarrow \sim a;\ c \leftarrow a\}$. If we interpret the meaning of this program as its WFM (which is empty), and as we do not have a, a naive CWA could be tempted to derive $\sim c$ based on the assumption $\sim a$. There is however an alternative negative assumption $\sim b$, that if made, defeats the assumption $\sim a$, i.e. the assumption $\sim a$ may not be sustained since it can be defeated by the assumption $\sim b$. We will define later more precisely the notions of sustainability and tenability.

Both programs above have empty well founded models. We argue that WFS is too careful, and something more can safely be added to the meaning of program, thus reducing the undefinedness of the program, if we are willing to adopt a suitable form of CWA.

We argue that a set $CWA(P)$ of negative literals (assumptions) *added* to a program model $MOD(P)$ by CWA must obey the four principles:

1. $MOD(P) \cup CWA(P) \not\models L$ for any $\sim L \in CWA(P)$. This says that the program model added with the set of assumptions identified by the CWA rule must be **consistent**.

2. There is no other set of assumptions A such that $MOD(P) \cup A \models L$ for some $\sim L \in CWA(P)$. I.e. $CWA(P)$ is **sustainable**.

3. $CWA(P)$ must be **maximal**.

4. $CWA(P)$ must be **unique**.

The paper is organized as follows: in the next section we present some basic definitions. In section 3 we introduce some new definitions, capturing the concepts behind the semantics, accompanied by examples illustrating them. Models are defined and organized into a lattice, and the class of sustainable A-Models is identified. In section 5 we define the O-Semantics of a program P based on the class of maximal sustainable tenable A-Models. A unique model is singled out as the O-Model of P. Afterwards we present some properties of the class of A-Models. Finally, we relate to other semantics and present conclusions.

2 Language

Here we give basic definitions and establish notation ([Monteiro, 1991]). A program is a set of rules of the form: $H \leftarrow B_1, \ldots, B_n, \sim C_1, \ldots, \sim C_m$ $(n \geq 0, m \geq 0)$ or equivalently $H \leftarrow \{B_1, \ldots, B_n\} \cup \sim\{C_1, \ldots, C_m\}$, where $\sim\{A_1, \ldots, A_n\}$ is a shorthand for $\{\sim A_1, \ldots, \sim A_n\}$, and $\sim C$ is short for $\sim\{C1, \ldots, C_m\}$; H,, B_i and C_j are atoms.

The Herbrand Base $B(P)$ of a program P is defined as usual as the set of all ground atoms. An interpretation I of P is denoted by $T \cup \sim F$, where T and F are disjoint subsets of $B(P)$. Atoms in T are said to be *true* in I, atoms in F *false* in I, and atoms in $B(P) - (T \cup F)$ *undefined* in I.

In an interpretation $T \cup \sim F$ a conjunction of literals $\{B_1, \ldots, B_n\} \cup \sim\{C_1, \ldots, C_m\}$ is *true* iff $\{B_1, \ldots, B_n\} \subseteq T$ and $\{C_1, \ldots, C_m\} \subseteq F$, is *false* iff $\{B_1, \ldots, B_n\} \cap F \neq \emptyset$ or $\{C_1, \ldots, C_m\} \cap F \neq \emptyset$, and is *undefined* iff it is neither true nor false.

3 Adding Negative Assumptions to a Program

Here we show how to consistently add negative assumptions to a program P. Informally, it is consistent to add a negative assumption to P if the assumption atom is not among the consequences P after adding the assumption. We also define when a set of negative assumptions is defeated by another, and show how the models of a program, for different sets of negative assumptions added to it, are organized into a lattice.

We begin by defining what it means to add assumptions to a program. This is achieved by substituting *true* for the assumptions, and *false* for their atoms, in the body of all rules.

Definition 3.1 (P+A) The program $P + A$ obtained by adding to a program P a set of negative assumptions $A \subseteq \sim B(P)$ is the result of:

- Deleting all rules $H \leftarrow \{B_1, \ldots, B_n\} \cup \sim C$ from P, such that some $B_i \in A$
- Deleting from the remaining rules all $\sim L \in A$

Definition 3.2 (Assumption Model) An Assumption Model of a program P, or A-Model for short, is a pair $\langle A; M \rangle$ where $A \subseteq \sim B(P)$ and $M = WFM(P + A)$.

Among these models we define the partial order $\leq_a$ in the following way: $\langle A_1; M_1 \rangle \leq_a \langle A_2; M_2 \rangle$ iff $A_1 \subseteq A_2$. On the basis of set union and set intersection among the sets A of negative assumptions, the set of all A-Models becomes organized as a complete lattice.

Having defined assumption models we next consider their consistency. According to the CWA principles above, an assumption $\sim A$ cannot be added to a program P if by doing so A is itself a consequence of P, or some other assumption is contradicted.

Definition 3.3 (Consistent A-Model) An A-Model $\langle A; M \rangle$ is consistent iff $A \cup M$ is an interpretation, i.e. there exists no assumption $\sim L \in A$ such that $L \in M$.

Example 1 Let $P = \{c \leftarrow\sim b;\ b \leftarrow\sim a;\ a \leftarrow\sim a\}$, whose WFM is empty. The A-Model $\langle\{\sim a\};\{a,b,\sim c\}\rangle$ is inconsistent since by adding the assumption $\sim a$ then $a \in WFM(P+\{\sim a\})$. The same happens with all A-Models containing the assumption $\sim a$. The A-Model $\langle\{\sim b,\sim c\};\{c\}\rangle$ is also inconsistent. Thus the only consistent A-Models are $\langle\{\};\{\}\rangle$, $\langle\{\sim b\};\{c\}\rangle$ and $\langle\{\sim c\};\{\}\rangle$. $\square$

Lemma 3.1 *If an A-Model AM is inconsistent then any AM' such that $AM \leq_a AM'$ is inconsistent.*

Proof:[sketch] We prove that for all $\sim a' \in B(P)$, if $\langle A; WFM(P+A)\rangle$ is inconsistent then $\langle A \cup \{\sim a'\}; WFM(P+A\cup\{\sim a'\})\rangle$ is also inconsistent. By definition of consistent A-Model: $\exists \sim b \in A \mid b \in WFM(P+A)$, so it suffices to guarantee that : $b \notin WFM(P+A\cup\{\sim a'\}) \rightarrow a' \in WFM(P+A\cup\{\sim a'\})$.

Consider $b \notin WFM(P+A\cup\{\sim a'\})$. Since $P+A\cup\{\sim a'\}$ only differs from $P+A$ in rules with a' or $\sim a'$, and since b is true in $P+A$, it can be shown a' is also true in $P+A$. As the truth of an atom in the WFM of any program may not rely neither on the truth of itself nor of its complementary, and because the addition of $\sim a'$ to $P+A$ only changes rules with $\sim a'$ or a', the truth value of a' in $P+A\cup\{\sim a'\}$ remains the same, i.e. $a' \in WFM(P+A\cup\{\sim a'\})$. $\diamondsuit$

According to the CWA principles above, an assumption $\sim A$ cannot be sustained if there is some set of consistent assumptions that concludes A. We've already expressed the notion of consistency being used. To capture the notion of sustainability we now formally define how an A-Model can defeat another, and define sustainable A-Models as the nondefeated consistent ones.

Definition 3.4 (Defeating)
A consistent A-Model $\langle A; M\rangle$ is defeated by a consistent $\langle A'; M'\rangle$ iff $\exists \sim a \in A | a \in M'$.

Definition 3.5 (Sustainable A-Models)
An A-Model $\langle A; M\rangle$ is sustainable iff it is consistent and not defeated by any consistent A-Model. Equivalently $\langle\sim S; M\rangle$ is sustainable iff:

$$S \cap \bigcup_{consistent\ \langle A_i;M_i\rangle} M_i = \{\}$$

Example 2 The only sustainable models in example 1 are $\langle\{\};\{\}\rangle$ and $\langle\{\sim b\};\{c\}\rangle$. Note that the consistent A-Model $\langle\{\sim c\};\{\}\rangle$ is defeated by $\langle\{\sim b\};\{c\}\rangle$, i.e. the assumption $\sim c$ is unsustainable since there is a set of consistent assumptions (namely $\{\sim b\}$) that leads to the conclusion c. $\square$

The assumptions part of maximal sustainable A-Models of a program P are maximal sets of consistent Closed World Assumptions that can be safely added to the consequences of P without risking contradiction by other assumptions.

Lemma 3.2 *If an A-Model AM is defeated by another A-Model D, then all A-Models AM' such that $AM \leq_a AM'$ are defeated by D.*

Proof: Similar to the proof of lemma 3.1 above. $\diamondsuit$

Lemma 3.3 *The A-Model $\langle\{\}; WFM(P)\rangle$ is always sustainable.*

Proof: By definition of sustainable. $\diamondsuit$

Theorem 3.4 *The set of all sustainable A-Models is nonempty. On the basis of set union and set intersection among its A sets, the A-Models ordered by $\leq_a$ form a lower semi lattice.*

Proof: Follows directly from the above lemmas. $\diamondsuit$

A program may have several maximal sustainable A-Models.

Example 3 Let
$P = \{c \leftarrow\sim c, \sim b;\ b \leftarrow a;\ a \leftarrow\sim a\}$. Its sustainable A-Models are $\langle\{\};\{\}\rangle$, $\langle\{\sim b\};\{\}\rangle$ and $\langle\{\sim c\};\{\}\rangle$. The last two are maximal sustainable A-Models. We cannot add both $\sim b$ and $\sim c$ to the program to obtain a sustainable A-Model since $\langle\{\sim b,\sim c\};\{c\}\rangle$ is inconsistent. $\square$

4 The O-semantics

This section is concerned with the problem of singling out, among all sustainable A-Models of a program P, one that uniquely determines the meaning of P when the CWA is enforced. This is accomplished by means of a selection criterium that takes a lower semilattice of sustainable A-Models and obtains a subsemilattice of it, by deleting A-Models that in a well defined sense are less preferable, i.e. the untenable ones.

Sustainability of a consistent set of negative assumptions insists that there be no other consistent set that defeats it (i.e. there is no hypothetical evidence whose consequences contradict the sustained assumptions). Tenability requires that a maximal sustainable set of assumptions be not contradicted by the consequences of adding to it another competing (nondefeating and nondefeated) maximal sustainable set.

The selection process is repeated and ends up with a complete lattice of sustainable A-Models, which defines for every program P its O-Semantics. The meaning of P is then specified by the greatest A-Model of the semantics, its O-Model.

To illustrate the problem of preference among maximal A-Models we give an example.

Example 4
Let $P = \{c \leftarrow \sim c, \sim b;\ b \leftarrow a;\ a \leftarrow \sim a\}$, whose sustainable A-Models are $\langle\{\};\{\}\rangle$, $\langle\{\sim b\};\{\}\rangle$, and $\langle\{\sim c\},\{\}\rangle$. Because we wish to maximize the number of negative assumptions we consider the maximal A-Models, which in this case are the last two. The join of these maximal A-Models, $\langle\{\sim b, \sim c\};\{c\}\rangle$, is per force inconsistent, in this case wrt c. This means that when assuming $\sim c$ there is an additional set of assumptions entailing c, making this A-Model *untenable*. But the same does not apply to $\sim b$. Thus the preferred A-Model is $\langle\{\sim b\},\{\}\rangle$, and the A-Model $\langle\{\sim c\};\{\}\rangle$ is said untenable. The rationale for the preference is grounded on the fact that the inconsistency of the join arises wrt c but not wrt b. □

Definition 4.1 (Candidate Structure)
A Candidate Structure CS of a program P is any subsemilattice of the lower semi lattice of all sustainable A-Models of P.

Definition 4.2 (Untenable A-Models)
Let $\{\langle A_1; M_1\rangle, \ldots, \langle A_n; M_n\rangle\}$ be the set of all maximal A-Models in Candidate Structure CS. Let $J = \langle A_J; M_J\rangle$ be the join of all such A-Models, in the complete lattice of all A-Models . An A-Model $\langle A_i; M_i\rangle$ is untenable wrt CS iff it is maximal in CS and there exists $\sim a \in A_i$ such that $a \in M_J$.

Proposition 4.1 *There exists no untenable A-Model wrt a Candidate Structure with a single maximal element.*

Proof: Since the join coincides with the unique maximal A-Model, which is sustainable by definition of CS, then it cannot be untenable. ◊

The Candidate Structure left after removing all untenable A-Models of a CS, may itself have several maximal elements, some of which might not be maximal A-Models in the initial CS. If the removal of untenable A-Models is performed repeatedly on the retained Candidate Structure, a single maximal element is eventually obtained, albeit the bottom element of all the CSs.

Definition 4.3 (Retained CS) *The Retained Candidate Structure $R(CS)$ of a Candidate Structure CS is:*

- *CS if it has a single maximal A-Model, i.e. CS is a complete lattice.*
- *Otherwise, let Unt be the set of all untenable A-Models wrt CS. Then $R(CS) = R(CS - Unt)$.*

Definition 4.4 (The O-Semantics)
The O-Semantics of a program P is defined by the Retained Candidate Structure of the semilattice of all sustainable A-Models of P.

Let $\langle A; M\rangle$ be its maximal element. The intended meaning of P is $A \cup M$, the O-Model of P.

Theorem 4.1 (Existence of O-Semantics)
The Retained Candidate Structure of the semilattice of all sustainable A-Models is nonempty.

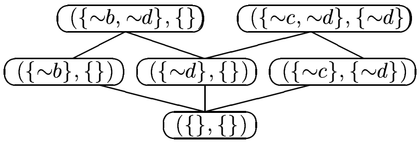
Proof:[sketch] It suffices to guarantee that at each iteration with more than one maximal A-Model at least one is untenable. This is done by contradiction: suppose no maximal A-Model is untenable. Then their join would be the single maximal sustainable one, and so could not be untenable, in the previous and final iteration; accordingly the supposed models cannot be maximal.

When there is a single maximal A-Model then the structure is a complete lattice, since at each iteration only maximal A-Models were removed. This lattice is nonempty since its bottom $\langle\{\}; WFM(P)\rangle$ is always sustainable and can never be untenable. ◊

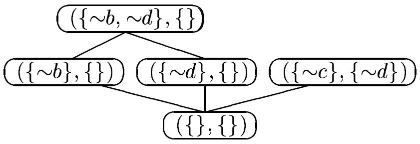
5 Examples

In this section we display some examples and their O-Semantics. Remark that indeed the O-Models obtained express the safe CWAs compatible with the WFMs (which are all $\{\}$).

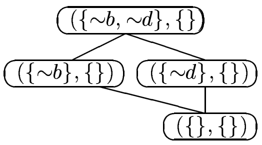
Example 5
Let $P = \{a \leftarrow \sim a;\ b \leftarrow a; c \leftarrow \sim c, \sim b;\ d \leftarrow c\}$ The semilattice of all sustainable A-Models CS is:



The join of its maximal A-Models is $\langle\{\sim b, \sim c, \sim d\}; \{c, \sim d\}\rangle$. Consequently, the maximal A-Model on the right is untenable since it contains $\sim c$ in the assumptions, and c is a consequence of the join. So $R(CS) = R(CS')$ where CS' is:



The join of all maximal elements in CS' is the same as before and the only untenable A-Model is again the maximal one having $\sim c$ in its assumptions. Thus $R(CS) = R(CS'')$ where CS'' is:



So the O-Model is $\{\sim b, \sim d\}$. Note that if P is divided into $P_1 = \{c \leftarrow \sim c, \sim b;\ d \leftarrow c\}$ and $P_2 = \{a \leftarrow \sim a;\ b \leftarrow a\}$, the O-models of P_1 and P_2 both agree on the only common literal $\sim b$. So $\sim b$ rightly belongs to the O-models of P. $\square$

Example 6 Let $P = \{q \leftarrow \sim p;\ p \leftarrow a;\ a \leftarrow \sim b;\ b \leftarrow \sim c;\ c \leftarrow \sim a\}$. Its only consistent A-Models are $\langle\{\}; \{\}\rangle$, $\langle\{\sim p\}; \{q\}\rangle$ and $\langle\{\sim q\}; \{\}\rangle$. As this last one is defeated by the second, the only sustainable ones are the first two. Since only one is maximal, these two A-Models determine the O-Semantics, and the meaning of P is $\{\sim p, q\}$, its O-Model. Note that if the three last rules, forming an *"undefined loop"*, are replaced by another *"undefined loop"* $a \leftarrow \sim a$, the O-model is the same. This is as it should, since the first two rules conclude nothing about a. $\square$

Example 7 Let $P = \{p \leftarrow a, b;\ a \leftarrow \sim b;\ b \leftarrow \sim a\}$. The A-Models with $\sim b$ in their assumptions defeat A-Models with $\sim a$ in their assumptions and vice-versa. Thus the O-Semantics is determined by $\langle\{\}; \{\}\rangle$ and $\langle\{\sim p\}; \{\}\rangle$, and the meaning of P is $\{\sim p\}$, its O-Model. $\square$

Example 8 Let $P = \{c \leftarrow \sim c, \sim b;\ b \leftarrow \sim c, \sim b;\ b \leftarrow a;\ a \leftarrow \sim a\}$. Its sustainable A-Models are $\langle\{\}; \{\}\rangle$, $\langle\{\sim b\}; \{\}\rangle$ and $\langle\{\sim c\}; \{\}\rangle$. The join of the maximal ones is $\langle\{\sim b, \sim c\}; \{b, c\}\rangle$, and so both are untenable. Thus the Retained Candidate Structure has the single element $\langle\{\}; \{\}\rangle$ and the meaning of P is $\{\}$ $\square$

6 Properties of Sustainable A-Models

This section explores properties of sustainable A-Models that provide a better understanding of them, and also give hints for their construction without having to previously calculate all A-Models.

We begin with properties that show how our models can be viewed as an extension to Well Founded Semantics (WFS). As mentioned in [Kakas and Mancarella, 1991a], negation in WFS is based on the notion of support, i.e. a literal $\sim L$ only belongs to an Extended Stable Model (XSM) if all the rules for L (if any) have false bodies in the XSM. In contradistinction, we are interested in negations as consistent hypotheses that cannot be defeated. To that end we weaken the necessary (but not sufficient) conditions for a negative literal to belong to a model as explained below. We still want to keep the necessary and sufficient conditions of support for positive literals. More precisely, knowing that XSMs must obey, among others, the following conditions cf. [Monteiro, 1991]:

- If there exists a rule $p \leftarrow B$ in the program such that B is true in model M then p is also true in M (*sufficiency of support for positive literals*).
- If an atom $p \in M$ then there exists a rule $p \leftarrow B$ in the program such that B is true in M (*necessity of support for positive literals*).
- If all rule bodies for p are false in M then $\sim p \in M$ (*sufficiency of support for negative literals*).
- If $\sim p \in M$ then all rules for p have false bodies in M (*necessity of support for negative literals*).

Our consistent A-models, when understood as the union of their pair of elements, assumptions A and $WFM(P+A)$, need not obey the fourth condition. Foregoing it condones making negative assumptions. In our models an atom might be false even if it has a rule whose body is undefined. Thus, only false atoms with an undefined rule body are candidates for having their negation added to the $WFM(P)$.

Proposition 6.1 *Let $\langle A; M\rangle$ be any consistent A-Model of a program P. The interpretation $A \cup M$ obeys the first three conditions above.*

Proof: Here we prove the satisfaction of the first condition. The remaining proofs are along the same lines.

If $\exists p \leftarrow b_1, \ldots, b_n, \sim c_1, \ldots, \sim c_m \in P \mid \{b_1, \ldots, b_n, \sim c_1, \ldots, \sim c_m\} \subseteq A \cup M$ then $b_i \in M$ $(1 \leq i \leq n)$ and $\sim c_j \in M$ or $\sim c_j \in A$ $(1 \leq j \leq m)$. Let $p \leftarrow b_1, \ldots, b_n, \sim c_l, \ldots, \sim c_k (l \geq 1, k \leq m)$ be the rule obtained from an existing one by removing all $\sim c_j \in A$, which is, by definition, a rule of $P+A$. Thus there exists a rule $p \leftarrow B$ in $P+A$ such that $B \subseteq WFM(P+A) = M$. Given that the WFM of any program must obey the first condition above, $p \in WFM(P+A)$. $\diamondsuit$

Next we state properties useful for more directly finding the sustainable A-Models.

Proposition 6.2 *There exists no consistent A-Model $\langle A; M\rangle$ of P with $\sim a \in A$ such that $a \in WFM(P)$.*

Proof: Let $\langle A; M\rangle$ be an A-Model such that $\sim a \in A$ and $a \in WFM(P)$. It is known that the truth of any $a \in WFM(P)$ cannot be supported neither on itself nor on $\sim a$. If $A = \{\sim a\}$ then, after adding $\{\sim a\}$ to the program, the rules supporting the truth of a remain unchanged, i.e. $a \in WFM(P + \{\sim a\})$, and thus $\langle\{\sim a\}; WFM(P + \{\sim a\})\rangle$ is inconsistent. It follows, from lemma 3.1, that all A-Models $\langle A; M\rangle$ such that $\{\sim a\} \subseteq A$ are inconsistent. $\diamondsuit$

Hence, A-Models not obeying the above restriction are not worth considering as sustainable.

Proposition 6.3
If a negative literal $\sim L \in WFM(P)$ then there is no consistent A-Model $\langle A; M\rangle$ of P such that $L \in M$.

Proof:[sketch] We prove that if $L \in M$ for a given A-Model $\langle A; M\rangle$ of P then $\langle A; M\rangle$ is inconsistent. If $L \in M$ there must exist a rule $L \leftarrow B, \sim C$ in P such that $B \cup \sim C \subseteq M \cup A$ and $B \cup \sim C$ is false in $WFM(P)$, i.e. there must exist $L \leftarrow B, \sim C$ in P with at least one body literal true in $M \cup A$ and false in $WFM(P)$. If that literal is an element of $\sim C$, by proposition 6.2, $\langle A; M\rangle$ is inconsistent (its corresponding atom is true in $WFM(P)$ and false in $M \cup A$). If it is an element of B this theorem applies recursively, ending up in a rule with empty body, an atom with no rules or a loop without an interposing $\sim l$. By definition of $WFM(P+A)$ the truth value of literals in these conditions can never be changed. $\diamondsuit$

Theorem 6.1 *If $\sim L \in WFM(P)$ then $\sim L \in M$ in every consistent A-Model $\langle A; M\rangle$ of P.*

Proof: Given proposition 6.3, it suffices to prove that L is not undefined in any consistent A-Model of P. The proof is along the lines of that of the proposition above. $\diamondsuit$

Consequently, all supported negative literals in the $WFM(P)$, which includes those without rules for their atom, belong to every sustainable A-Model.

Lemma 6.2 *Let $WFM(P) = T \cup \sim F$. For any subset S of $\sim F$, $WFM(P) = WFM(P+S)$.*

Proof: This lemma is easily shown using the definition of $P+A$ and the properties of the WFM. $\diamondsuit$

Theorem 6.3 *Let $WFM(P) = T \cup \sim F$ and $\langle A; WFM(P+A)\rangle$ be a consistent A-Model, and let $A' = A \cap \sim F$. Then $WFM(P+A) = WFM(P+(A-A'))$.*

Proof: Let $P' = P + (A - A')$, and $WFM(P) = T \cup \sim F$. By theorem 6.1 $\sim F \subseteq WFM(P')$. So, by lemma 6.2, $WFM(P') = WFM(P' + \sim F) = WFM([P+(A-(A \cap \sim F))] + \sim F)$. By definition of $P+A$ it follows that $(P+A_1)+A_2 = P+(A_1 \cup A_2)$. Thus $WFM(P')$ is:

$$\begin{aligned} & WFM(P + [(A - (A \cap \sim F) \cup \sim F]) \\ = \; & WFM(P+A) \end{aligned}$$

$\diamondsuit$

This theorem shows that sets of assumptions including negative literals of $WFM(P)$ are not worth considering since there exist smaller sets having exactly the same consequences $A \cup M$ and, by proposition 6.3 the larger sets are not defeatable by reason of negative literals from the $WFM(P)$.

Another important hint for calculating the sustainable A-Models is given by lemma 3.1. According to it one should start by calculating A-Models with smaller assumption sets, so that when an inconsistent A-Model is found, by the lemma, sets of assumptions containing it are unworth considering.

Example 9
Let $P = \{p \leftarrow \sim a, \sim b;\ a \leftarrow c, d;\ c \leftarrow \sim c;\ d\}$. The least A-Model is $\langle \{\}; \{d, \sim b\} \rangle$ where $\{d, \sim b\} = WFM(P)$. Thus sets of assumptions containing $\sim d$ or $\sim b$ are not worth considering. Take now, for example, the consistent A-Model $\langle \{\sim a\}; \{d, \sim b, p\} \rangle$, which we retain. Consider $\langle \{\sim c\}; \{c, a, \sim p\} \rangle$; as this A-Model is inconsistent we do not retain it nor consider any other A-Models with assumption sets containing $\sim c$. Now we are left with just two more A-Models worth considering: $\langle \{\sim p\}; \{d, \sim b\} \rangle$ which is defeated by $\langle \{\sim a\}; \{d, \sim b, p\} \rangle$; and $\langle \{\sim p, \sim a\}; \{d, \sim b, p\} \rangle$ which is inconsistent. Thus the only two sustainable A-Models are $\langle \{\}; \{d, \sim b\} \rangle$ and $\langle \{\sim a\}; \{d, \sim b, p\} \rangle$. In this case, the latter is the single maximal sustainable A-Model, and thus uniquely determines the intended meaning of P to be $A \cup M = \{\sim a, d, \sim b, p\}$. $\Box$

7 Relation to other work

Consider the following program ([Van Gelder *et al.*, 1980]):

$$\begin{aligned} P \;=\; & \{p \leftarrow q, \sim r, \sim s;\ q \leftarrow r, \sim p; \\ & r \leftarrow p, \sim q;\ s \leftarrow \sim p, \sim q, \sim r\} \end{aligned}$$

In [Przymusinska and Przymusinski, 1990] they argue that the intended semantics of this program should be the interpretation $\{s, \sim p, \sim q, \sim r\}$ due to the mutual circularity of p, q, r. This model is precisely the meaning assigned to the program by the O-Semantics, its O-Model. Note that WFS identifies the (3-valued) empty model as the meaning of the program. This is also the model provided by stable model semantics [Gelfond and Lifschitz, 1988]. The weakly perfect model semantics for this program is undefined as noticed in [Przymusinska and Przymusinski, 1990].

The EWFS [Baral *et al.*, 1990] is also an extension to the WFM based on the notion of GCWA [Minker, 1987]. Roughly EWFS moves closer than the WFM (in the sense of being less undefined) to being the intersection of all minimal Herbrand models of P [Dix, 1991]:

$$\begin{gathered} EWFM(P) =_{def} \\ WFM(P) + \langle T(WFM(P)), F(WFM(P)) \rangle \end{gathered}$$

where: $T(I) =_{def} True(I - MIN - MOD(P))$, $F(I) =_{def} False(I - MIN - MOD(P))$ and $I{-}MIN{-}MOD(P)$ is the collection of all minimal models consistent with the three valued interpretation I.

For the program $P = \{a \leftarrow \sim a\}$ we have:

$$\begin{gathered} WFM(P) = \{\}, \\ MIN{-}MOD(P) = \{a\} \text{ and } EWFM(P) = \{a\} \end{gathered}$$

Note this view identifies the intended meaning of rule $a \leftarrow \sim a$ as the equivalent logic formula $a \leftarrow \neg a$, i.e. a. The O-Model of P is empty.

The difference between the O-Semantics and EWFS may be noticed in the intended meaning of the two rule program: $\{a \leftarrow \sim b;\ b \leftarrow \sim a\}$, which is behind the motivation of the extension EFWS of WFM based on GCWA. EWFS wants to identify $a \vee b$ as the meaning of this program, which also justifies the identification of $a \leftarrow \sim a$ with the fact a. The O-Model is empty.

A similar approach based on the notion of stable negative hypotheses (built upon the notion of consistency) is introduced in [Kakas and Mancarella, 1991b], identifying a stable theory associated with a program P as a *"skeptical"* semantics for P, that always contains the well founded model.

One example showing that their approach is still conservative is: $\{p \leftarrow \sim q;\ q \leftarrow \sim r;\ r \leftarrow \sim p;\ s \leftarrow p\}$. Stable theories identifies the empty set as the meaning of the program; however its O-Model is $\{\sim s\}$, since it is consistent, maximal, sustainable and tenable. Kakas (personnal communication) now also obtains this model, as a result of the investigation mentioned in the conclusions of [Kakas and Mancarella, 1991b].

8 Conclusions

We identify the meaning of a program P as a suitable partial closure of the well founded model of the program in the sense that it contains the well founded model (and thus always exists). The extension we propose reduces undefinedness (which some authors argue is a desirable property) in the intended meaning of a program P, by an adequate form of CWA based on notions of consistency, sustainability and tenability with regard to alternative negative assumptions. Sustainability of a consistent set of negative assumptions insists that there be no other consistent set that defeats it (i.e. there is no hypothetical evidence whose consequences contradict the sustained assumptions). Tenability requires that a maximal sustainable set of assumptions be not contradicted by the consequences of adding to it another competing (nondefeating and nondefeated) maximal sustainable set.

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