Avoiding or Restricting Defectors in Public Goods Games?

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Abstract

When creating a public good, strategies or mechanisms are required to handle defectors. We first show mathematically and numerically that prior agreements with posterior compensations provide a strategic solution that leads to substantial levels of cooperation in the context of Public Goods games, results that are corroborated by available experimental data. Notwithstanding this success, one cannot, as with other approaches, fully exclude the presence of defectors, raising the question of how they can be dealt with to avoid the demise of the common good. We show that both avoiding creation of the common good, whenever full agreement is not reached, and limiting the benefit that disagreeing defectors can acquire, using costly restriction mechanisms, are relevant choices. Nonetheless, restriction mechanisms are found the more favorable, especially in larger group interactions. Given decreasing restriction costs, introducing restraining measures to cope with public goods free-riding issues is the ultimate advantageous solution for all participants, rather than avoiding its creation.

Keywords: evolutionary games, cooperation, commitment, public goods.

1 Introduction

Arranging a prior commitment or agreement is an essential ingredient to encourage cooperative behavior in a wide range of relationships, ranging from personal to political and religious ones [1–5]. Prior agreements clarify the intentions and preferences of other players. Hence, refusing to establish an agreement may be considered as intending or preferring not to cooperate (non-committers) [5–7]. Prior agreements may be highly rewarding in group situations, as in the case of Public Goods Games (PGG) [8], as it forces the other participants to signal their willingness to achieve a common goal. Especially for increasing group sizes, such prior agreements could be ultimately rewarding, as it becomes more and more difficult to assess the aspirations of all participants.

In a PGG, where players meet in groups of size N [9,10], all players can choose whether to cooperate and contribute an amount, c, to the public good or to defect and take advantage of the public good without contributing to it. The total contribution is multiplied by a constant public goods producing factor, $r > 1$, and the result is afterwards distributed equally among all players. With r smaller than the group size ($r < N$), non-contributing free-riders always gain more than contributors. Evolutionary game dynamics has shown that under those conditions cooperation disappears, which is famously known as the 'tragedy of commons' [10,11]. Various mechanisms, such as direct and indirect reciprocity, kin and group selections, and costly punishment, have been proposed and evaluated both theoretically and experimentally, which explain the evolution of cooperation nevertheless [10,12,13], ranging from microbial systems to animals and humans societies [12,14–18].

Here, we examine a strategic solution based on prior agreements to address the problem of the evolution of cooperation in PGG. Prior to the PGG, commitment proposing players ask their co-players to commit to contribute to the PGG, paying a personal proposer's cost to establish that agreement. If all the requested co-players accept the commitment, the proposers assume that everyone will contribute to the public good. Those individuals that commit yet later do not contribute receive a penalty and are forced to compensate the proposers at a cost[6,19,20]. As such, our model explicitly and novelly addresses the relevance of the commitment proposing behavior regarding posterior compensations in group interactions, which has been suggested to be a major pathway to the emergence of cooperation[1,5].

As commitment proposers may encounter also non-committers, they require strategies that can deal with this kind of individuals[1,6,21]. When dealing with non-committers, the simplest strategy is to not participate in the creation of the common good or, when the interaction is mandatory, to simply not contribute, i.e. defect[9,22], when not everyone commits. Yet, this *avoidance* strategy also removes the benefits for those that wished to establish the public good, hindering any advancements they could harness from this novel resource. Alternatively, one can try to establish boundaries on the common good so that only those that commit to make it work have access or that the benefit non-contributors can acquire from the common good is reduced, as is the case for food sharing, aid in social health and defence against predators[23–25]. An extreme case of exercising restriction is ostracism, which can be enforced through financial or social means[26,27]. Experimental studies with PGG have shown that the threat of excluding or ostracizing non-cooperative members from the PGG can significantly increase contribution and cooperation[28,29]. As public goods are by definition non-excludable[8,30], ostracism may not be possible and non-committers may only be excluded to a certain degree. Moreover, a cost may be associated with this *exclusion* strategy, where the capacity to ostracize may be too costly. Evidence regarding restriction abounds in biological and social contexts: Animals fence and defend territory and resources[25]. Trade restrictions against non-participating countries are widely implemented in international and environmental treaties[8,31], yet may be circumvented.

While showing the relevance of our prior conclusions on commitment obtained for the pairwise prisoner's dilemma (PD)[20] within the context of the more complex PGG, we focus here on showing mathematically and numerically how best to deal with individuals that do not wish to make prior agreements and do not contribute to the common good. This issue is not only essential in the general discussion of the PGG, it is also fundamental in case of the strategic commitment behavior since we observed in the PD that the number of non-committers, together with those that free-ride on the investment of committers, increase markedly with the increase of the cost of setting up the commitment[20]. We will examine under which conditions avoidance, which is a generalization of the PD commitment behavior towards the PGG, and restricting strategies are beneficial in the PGG, determining at the same time the condition when the latter strategy is preferred over the former. The effect of the different parameters implicit to the strategies on their viability is carefully analyzed. Interestingly, we will show that group size is an important factor in determining the conditions for which restriction may be better than simply avoiding non-committers.

2 Results

2.1 Commitment strategies in PGG

Commitment strategies can propose a commitment deal to all members of the group before playing the PGG. The proposer(s) share the cost ϵ_P, while those who do not (but still can join the commitment) pay nothing. If all the requested co-players agree to the commitment, they are assumed to contribute to the public good. Those that commit though later do not contribute have to compensate the commitment proposers at their personal cost δ, and their compensation is shared among all proposers. Additionally, there may be group members that refuse the agreement, wishing to play the game without any prior commitment. As refusal may be conceived as a future defection (no contribution), proposers may wish to either avoid interacting with them or set up some mechanism that restricts their access to the public good. We define those strategies as follows:

- AVOID: refuses to play the game when there are non-committers in the group (hence, the PGG does not take place and each player receives 0 payoff).

- RESTRICT: sets up, at an extra cost ϵ_R, a mechanism to restrict the access of the non-committers to the public good. This restriction is modeled through a factor, $\psi < 1$, representing the fraction of the common resource the non-committers receive compared to the committed players (i.e. the smaller ψ the greater the effect).

Next to the traditional unconditional contributors (C, who always commit when being proposed a commitment deal, contribute whenever the PGG is played, but do not propose commitment) and unconditional non-contributors (D, who do not accept commitment, defect when the PGG is played, and do not propose commitment), we consider two commitment free-riding strategies, which we have shown to become dominant under certain conditions in the pair-wise PD situation[20]: (i) fake committers (FAKE), who accept a commitment proposal yet do not contribute whenever the PGG is played. These players assume that they can exploit the commitment proposing players without suffering the consequences; (ii) commitment free-riders (FREE), who defect unless being proposed a commitment, which they then accept and cooperate subsequently in the PGG. In other words, these players are willing to contribute when a commitment is proposed but are not prepared to pay the cost of setting it up.

We consider a well-mixed, finite population of a constant size Z, potentially composed of those five strategies, i.e. AVOID (or RESTRICT), C, FREE, D, and FAKE. To simplify the notations, the strategies are numerated 1, 2, 3, 4, and 5, respectively (where 1 can either indicate the AVOID or RESTRICT strategy). In each interaction, N individuals are randomly selected from the population for playing the PGG. Among N randomly selected players, the number of players in the group of size N using strategy i is denoted by $N_i,\ i = 1, \dots, 5$, such that $N = N_1 + N_2 + N_3 + N_4 + N_5$.

We compute the payoffs of either the AVOID or RESTRICT strategy in the population in relation to the other strategies (see Methods and Supporting Information (SI)). In case of the AVOID strategy, if there is a D player in the group, i.e. $N_4 \geq 1$, the game is not played and every player in the group obtains 0 (the results will be unchanged if the game is not optional, as in that case, AVOID players contribute nothing to the public good). In case of the RESTRICT strategy, the game is always played and RESTRICT players share an additional cost, $\epsilon_R > 0$, to restrict the benefits D players can obtain from the public good. The committing player (RESTRICT, C, FREE and FAKE) and non-committing player (D) in the PGG gain respectively $\frac{r(N_1+N_2+N_3)}{N_1+N_2+N_3+\psi N_4+N_5}c$ and $\frac{r(N_1+N_2+N_3)\psi}{N_1+N_2+N_3+\psi N_4+N_5}c$. The gain for a RESTRICT player is reduced by $c + \frac{1}{N_1}\epsilon_P - \frac{N_5}{N_1}\delta$, if $N_4 = 0$, and by $c + \frac{1}{N_1}(\epsilon_P + \epsilon_R) - \frac{N_5}{N_1}\delta$, otherwise. The payoff for a C and a FREE player is reduced by c. Finally, the payoff for a FAKE player is reduced by δ. The detailed calculation of the payoff for each strategy as well as the payoff matrix is provided in SI.

2.2 Constraints for viability of AVOID and RESTRICT

We derive the conditions for which commitment strategies, AVOID and RESTRICT, are evolutionary viable in a PGG, showing when they are risk-dominant (see Methods) against all defectors and free-riders (i.e. FAKE, FREE and D players). Yet more importantly, we determine when RESTRICT becomes more advantageous than AVOID; that is, when it is worthwhile to pay extra cost to invest in restriction technologies and infrastructure that limit the benefits of non-committers (D).

Equation (16) (see Methods) allows one to determine when AVOID and RESTRICT are risk-dominant against FREE. This occurs when

$$\sum_{k=1}^{N}(rc - c - \epsilon_P/k) \geq (N-1)(rc-c), \tag{1}$$

which can be simplified to

$$\epsilon_P \leq c(r-1)/F_N, \qquad \text{with } F_N = \sum_{k=1}^{N} 1/k \tag{2}$$

AVOID and RESTRICT are risk-dominant against FAKE if

$$\sum_{k=1}^{N}\left((\frac{rk}{N}-1)c + \frac{N\delta - \epsilon_P}{k} - \delta\right) \geq \sum_{k=1}^{N-1}\left(\frac{rk}{N}c - \delta\right). \tag{3}$$

Which can also be simplified to

$$\delta \geq \frac{N-r}{NF_{N-1}}c + \frac{F_N}{NF_{N-1}}\epsilon_P. \tag{4}$$

Now, as we aim to examine when restriction works better than avoiding non-committers, we first examine independently when AVOID or RESTRICT are risk-dominant agains D players. In case of the AVOID strategy this occurs when:

$$\epsilon_P \leq N(r-1)c. \tag{5}$$

As Equation (2) is more restrictive than Equation (5), the two conditions in Equations (2) and (4) define when AVOID is risk-dominant against all types of defectors and free-riders (see SI for simplifications of these formulas using inequalities for F_N). Both conditions can be understood intuitively. For a successful commitment, the cost of arranging the commitment needs to be justified with respect to the benefit of (mutual) cooperation (i.e. $r(c-1)$), and a compensation needs to be arranged (see Equation 4) that is proportional to the player's contribution and the investment cost she paid for setting up the commitment.

This observation becomes clearer when looking at the transition probabilities and stationary distribution in a population of AVOID players with the other four strategies, as shown in Figure 1a. Note the cycles from C to defection strategies (FREE, D and FAKE) and back to AVOID strategists, showing that defection strategies cannot completely be avoided in the PGG context (see also Figure S1): When the cost of arranging commitment, ϵ_P, is sufficiently small, the population spends most of the time in the homogeneous state with AVOID players, regardless of the initial composition of the population (Figure 1b). For low ϵ_P, nearly homogeneous AVOID populations are almost always reached for sufficiently large δ. More interestingly, this high frequency is not affected by changes in the compensation δ, once a certain threshold is reached. Accordingly, as for the PD[20], the arrangement cost is the essential parameter for the emergence and survival of AVOID and mutual cooperation. Additionally, we observe that for a variety of group sizes N, the region of ϵ_P wherein AVOID is a viable strategy increases (see Figure S3, considering also the fact that right hand side of Equation (2) is an increasing function of N): AVOID can handle the commitment free-riding strategies for a wider range of arrangement costs. Yet as the groups size increases, the frequency of AVOID, for similar small values of ϵ_P, decreases, revealing that other strategies may be necessary to cope with the increasing number of defectors in the groups and the population (Figure S3). These results provide a novel insight, when moving from the PD to the PGG, which is that avoiding defectors by refusing to play the game when someone does not agree to commit might lead to cooperation at higher arrangement costs, yet may in turn be detrimental for the overall level of cooperation in the game.

In turn, RESTRICT is risk-dominant against D when

$$\sum_{k=1}^{N-1} \frac{rkc(1-\psi)}{k+\psi(N-k)} \geq (N-r)c + F_N(\epsilon_P + \epsilon_R). \tag{6}$$

Because the left hand side of Equation (6) is a strictly decreasing function of ψ (see SI), the necessary condition for RESTRICT to be risk-dominant against D is (i.e. when $\psi = 0$)

$$(N-1)rc \geq (N-r)c + F_N(\epsilon_P + \epsilon_R), \tag{7}$$

which is equivalent to

$$\epsilon_P + \epsilon_R \leq \frac{N(r-1)}{F_N}c. \tag{8}$$

As the left hand side of Equation (6) is a continuous function of ψ, the satisfaction of Equation (8) guarantees that, for any given ϵ_P and ϵ_R, there exists a threshold ψ^D such that RESTRICT is risk-dominant against D for any ψ below it. This restriction threshold could be interpreted as the organizational or technological advancement required to guarantee success against individuals that exploit the non-exclusive public good. Moreover, it specifies what the limit is on the cost, combining the restriction and the proposing costs, for this to work.

These observations are supported by Figure 2a, where we show the transition probabilities and stationary distribution in a population of RESTRICT players with other non-commitment proposing strategies. The main difference with Figure 1a, is the increase in the fixation probability from D to RESTRICT. This effect depends on the value of restriction factor ψ, as is shown in Figure 2b. In general, the better the effect of restriction on non-committers (i.e. the smaller ψ), the higher the frequency of RESTRICT and cooperation in the long run, regardless of the initial composition of the population. These observations are robust against changes in the restriction ϵ_R, as seen in Figure 2c, where we show the frequency of RESTRICT varying both ψ and ϵ_R. One can observe in both figures that ψ is the decisive parameter on the frequency of RESTRICT: To achieve significantly high frequency of RESTRICT, and as a consequence cooperation, a stringent restriction of non-committers must be possible. Even when costless restriction (i.e. $\epsilon_R = 0$) is available, the frequency of RESTRICT decreases quickly when ψ

approaches 1. Notice that when $\psi = 1$, RESTRICT is never risk-dominant against D players, as can be seen from Equation (6).

This notable success of the RESTRICT strategy in dealing with non-committers becomes even more significant when the group size increases (see Figure 2d). When the cost of restriction is extreme (e.g. $\epsilon_R = 2$), D players dominate when the group size is small. But when the group size is sufficiently large, thereby reducing the individual cost of implementing the restriction, which is shared by the proposers, RESTRICT becomes dominant. The frequency of RESTRICT is even higher when ϵ_R is small (see already Figure 3b). It is also interesting to note that the necessary condition for RESTRICT to be risk-dominant against D, as specified in Equation (8), is simpler for larger N, since the right hand side of the equation is an increasing function of N (see SI).

2.3 What to do with the non-committers?

Since there is no difference between AVOID and RESTRICT, except when playing D, one only needs to compare the stationary distribution of AVOID in a population with only D players against the stationary distribution of RESTRICT in a similar population. One can show that the frequency of RESTRICT is greater than AVOID if and only if the ratio of transition probabilities from D to RESTRICT and vice versa is greater than the ratio of transition probabilities from D to AVOID and vice versa, which can further be simplified, in the large population limit, to[32] (see SI)

$$\sum_{k=1}^{N-1} \frac{rkc(1-\psi)}{k+\psi(N-k)} - (N-r)c - F_N(\epsilon_P + \epsilon_R) \geq rc - c - \epsilon_P/N, \tag{9}$$

which is equivalent to

$$\sum_{k=1}^{N-1} \frac{k(1-\psi)}{\psi N + k(1-\psi)} rc - F_N\epsilon_R - F_{N-1}\epsilon_P - (N-1)c \geq 0. \tag{10}$$

The left hand side of this equation is a decreasing function of ψ, ϵ_R and ϵ_P, but increasing in r (see proofs in SI). Note that the cost ϵ_P still persists because RESTRICT players need to pay this cost when there are D players in the group, while AVOID players do not have to pay it as they refuse to play with D players (thereby not arranging any commitment deal).

An observation that can be derived immediately from Equation (10) is that, when $\psi = 1$, the equation is not satisfied. That is, unless restriction is possible, at least to some degree, it is better to refuse to play with those who explicitly do not agree to commit. Furthermore, with $\psi = 0$ we obtain the necessary condition for RESTRICT to be favored over AVOID:

$$(r-1)c \geq \frac{F_N}{N-1}\epsilon_R + \frac{F_{N-1}}{N-1}\epsilon_P. \tag{11}$$

It means the cost of arranging the agreement and restricting the benefit of non-contributors needs to be justified with respect to the benefit of the PGG. Only in that case can restriction become justified over avoidance or non-participation. If this equation is not satisfied, AVOID is the better commitment strategy, however good the restriction of non-committers that can be brought about, including full exclusion or ostracism.

In addition, as the left hand side of Equation (10) is a continuous function of ψ, this equation guarantees that, for any given ϵ_P and ϵ_R, there exists a threshold ψ^{AVOID} which, for any $\psi < \psi^{AVOID}$, RESTRICT performs better than AVOID. This threshold moreover increases with r and can approach infinitely close to 1 when r tends to infinity, because, as mentioned above, the left hand side of this equation is a decreasing function of ψ, but increasing in r (see Figures S4 and S5 in SI). In addition, the equation provides the criteria for whether it is worthwhile to develop the restriction infrastructure or mechanism. As technological evolution may significantly (and unlimitedly) reduce the cost-to-impact ratio, for instance with respect to restriction mechanisms that require computing and communication power[33,34], one can postulate that under this condition RESTRICT will eventually be available and deployable, even when currently AVOID seems to be the best choice.

Figure 3a shows the region of parameters where RESTRICT is better than AVOID, generated from the analytical condition in Equation (10). There is a large range of costs for restricting the access of non-committers, ϵ_R, and the effect of restriction, ψ, where RESTRICT is better than AVOID. Yet, if ϵ_R is too large, even a full exclusion (ostracism) does not lead to a better commitment strategy (as also can be seen from Equation (11)). These analytical results are compatible with the numerical simulation results shown in Figure 2c (see SI for more comparisons, Figure S6).

In addition, because the right hand side of Equation (11) is a decreasing function of N, converging to 0 when N approaches infinity (see proofs in SI), AVOID is less preferred for increasing N: As group size increases (see Figure 3b), the frequency of the strategy decreases. This indicates that avoiding non-committers in larger groups is less successful than avoiding them in smaller ones. Interestingly, RESTRICT seems to cope better with changes in group size: As group size increases, RESTRICT becomes more frequent than AVOID for a larger value of the restriction cost. When the cost of restriction is small (e.g. $\epsilon_R = 0.25$), RESTRICT ensures a higher level of committers, and as a consequence contributors, in comparison to the AVOID strategy. Even when the restriction cost is high, there is a certain group size for which RESTRICT is more abundant than AVOID, see Figure 3b. Furthermore, as can be seen in Figures S4 and S5, when N increases, RESTRICT is the better strategy for strictly larger ranges of ϵ_P, ϵ_R and ψ, which becomes even more apparent for increasing values of the public good's multiplication factor r.

3 Discussion

We have shown that arranging prior commitment can lead to the evolution of cooperation in a PGG when the cost of arranging commitment is justified with respect to the benefit of cooperation. As such, this result generalizes the conclusions previously obtained for the PD[20], underlining again the evolutionary advantage of this capacity to make prior agreements in combination with the capacity to send signals and act accordingly[1,35]. Moreover, we show that even though the commitment strategies become viable for a wider range of the arrangement cost when moving from the PD to the PGG, and more generally, when the group size of the PGG increases, it might be detrimental for the overall level of cooperation when the cost of arrangement is low. Nonetheless, prior agreements remain more efficient in achieving cooperation when being compared to simple peer punishment[9,36] (see Figure S7) in the PGG, an important result we observed also in case of the PD scenario[20].

Notwithstanding this efficiency with respect to punishment, individuals that do not accept an agreement (D) or individuals that free-ride on the investment that commitment proposers make (FREE) may increase in frequency within both the PD and PGG contexts as the cost of setting up the commitment increases. We examined here in detail how to deal with the former: Either the commitment proposers can decide to avoid interacting with those non-committers (AVOID) or implement an infrastructure that allows them to restrict their access (RESTRICT), which corresponds to a reduction in the benefit they can obtain from the public good. For both strategies it is assumed that an institution is present in order to enforce the compensation when someone in the commitment does not honor it. As such, both strategies are closely related to pool punishment models in terms of the presence of a third party required for the execution of the process[9,22]. Yet AVOID and RESTRICT are also different from pool punishment as in the latter system no prior agreement on the posterior compensations is made. Additionally, the main difference between AVOID and RESTRICT, in terms of execution, is that, AVOID does not pay to set up the agreement when someone does not accept the agreement, whereas RESTRICT always initiates the agreement to play and pays an additional fee to the institution to reduce the access of the (explicit) non-committers.

We show here that for both strategies one can identify intuitive conditions, defined by costs and compensations, that lead to increased likelihood in receiving contributions for the common good. Furthermore, we have compared the AVOID and RESTRICT strategies and provided analytical conditions for when one mechanism is better than the other. Our results show that RESTRICT is better than AVOID if non-committers can be restricted to a certain degree, with a small enough cost. Otherwise, it is better to rely on AVOID, i.e. simply refuse to interact if there are non-committers in the group (or, if the interaction is mandatory, to not contribute when playing the PGG). The restriction effect ψ, which is defined as the reduction of the benefit of the non-committer, was shown to be a decisive factor. Interestingly, its threshold ψ^{AVOID} (i.e. for all $\psi < \psi^{AVOID}$, RESTRICT is better than AVOID) increases with r and can be infinitely close to 1. This indicates that for given costs and compensations of commitment, any restriction mechanism can be more advantageous than AVOID when the PGG is sufficiently beneficial.

Moving from pairwise to multi-player interactions requires an analysis of the role of the group size in relation to the viability of the commitment strategies. Indeed, as group size increases, RESTRICT becomes more viable than AVOID even when it is costlier to implement the restriction measure. Furthermore, it is so for a strictly larger range of the commitment costs and restriction factors, especially when the public good's producing factor (r) is high. As such, these results indicate that larger public goods with higher levels of contribution can be established once restriction becomes possible. The amount of restriction depends on the gain each participant gets from the public good: the lower the gain, the tougher the restriction needs to be.

These results differ from other observations related to the impact of group size on the level of cooperation in PGG[37,38]: Lehmann et al.[37] showed that if the group size can be expanded stochastically, for instance, as a result of an increase in fecundity and/or a decrease in mortality, the kin-competition pressure induced by the limited dispersal in their networked model, can be significantly reduced, thereby favoring the evolution of cooperative behavior. In a similar manner, Alizon and Taylor[38] showed that if the group size and compositions can be adapted over time, in a way that reduces the competition among relatives in a structured population (by allowing groups or patches with high fecundity rates to grow faster), the cooperation level is increased.

In contrast, the strategic mechanisms we examine here do not consider any forms of relatedness between group members or structured populations. The essential message is that for smaller groups one is better off to avoid individuals that do not wish to accept an agreement, prior to the game, when the cost of restriction is too large. Yet as group size increases, restriction mechanisms are more efficient in achieving cooperation, depending on the association with the restriction. As such these results provide a completely new perspective on the role of group size on the level of cooperation. We envisage that AVOID and RESTRICT may also be more efficient in a structured population because the free-riders can be avoided and excluded permanently by removing links with them. Furthermore, an introduction of relatedness among individuals may reduce the need for arranging commitments as it provides additional incentive to not free-ride. Both issues may be explored in subsequent papers.

The results presented here are in accordance with the outcomes of different behavioral commitment experiments[6,19,28,29]. High levels of cooperation were observed in a PGG experiment where a binding agreement, which was enabled through a prior communication stage among the members of the group, could be arranged before the PGG interaction occurs[6]. The experiment showed that whenever a commitment deal is not binding or not enforced, corresponding to a low compensation cost δ in our commitment models, defectors are widespread and the contribution level is low. Commitment can also take the form of a deposit-refund scheme[19], where those who agree to commit have to deposit an amount which will be refunded only if they fulfill the commitment and contribute to the public good. Similarly to our results for the AVOID strategy, the most successful commitment strategy in that work was shown to be the one that refuses to set up the public good whenever there is a non-committer in the group (RESTRICT was not considered). The outcome of this deposit-refund experiment showed that when the deposit amount, corresponding to δ in our model, is sufficiently high, the contribution level is significant[19]. Note, however, that in both these experiments[6,19], the cost of setting up the commitment is always set to 0, thereby leading to effortless and effective commitment strategies. But as we have shown, this cost is the decisive factor for the viability of commitments strategies. This said, despite the fact that commitment has been shown experimentally to be a successful strategy for promoting cooperation in the PGG, our results further the understanding of the mechanism by identifying under which region of the parameters' values the mechanism works (as in the experiments) and when it does not. As a result, the outcome of our analysis suggests the need to study how varying the parameters would affect the outcomes of those commitment experiments.

Furthermore, PGG experiments, where exclusion of disapproved members (for example, through voting) is allowed, exhibit a high level of contribution and commitment[28,29]. But therein exclusion is carried out after the PGG takes place, towards the observed non-contributors, as in the model of Sasaki and Uchida[39], which is different from our model where restriction occurs before the game takes place. This suggests that social exclusion or ostracism, even when it requires an additional cost and/or has a reduced effect in terms of the restriction, is an important mechanism for promoting group cooperation[26,29]. However, we envisage that exclusion imposed through arranging prior commitment as in this work may be more suitable in the case where there is rivalry in the game, as is the case for Common-pool Resource games[8], as in that case posterior restriction would not hinder the participating players in collecting their benefit.

Various extensions to the current model can be addressed. First, one can consider to move beyond the symmetric commitments, where the cost for arranging and managing the agreement is equally shared among the proposers. Asymmetric commitments, where the contribution to manage the agreement may depend on the wealth and the potential benefits of each member as in inequality models[40,41], may further increase the realism of the conclusions one can draw from these models. Moreover, when extending to the repeated interaction scenario[42], it is also natural to consider that those who benefited more from the previous interactions should contribute more to the management of the commitments. We envisage that these seemingly fairer ways of sharing the benefit and cost of commitment can elevate the willingness to commit and contribute. In this repeated interaction context, commitments can also be made incrementally, conditional on behaviors of others in the previous round of interaction; this option has been shown to promote a higher level of contribution in a repeated PGG experiment[43].

In summary, our results have demonstrated that arranging prior commitments provides an important pathway for the emerge of cooperation in the one-shot Public Good Games, suggesting that good agreements make good friends[20] also in group interactions. Furthermore, always avoiding to play with those unwilling to commit is detrimental for the overall level of contribution, especially when interacting in large groups, and restriction towards those players might provide a better path to enhance the contribution level.

4 Methods

4.1 Population setup and evolutionary dynamics

Both the analytical and numerical results obtained here use Evolutionary Game Theory methods for finite populations[10,44,45]. In such a setting, individuals' payoff represents their *fitness* or social *success*, and evolutionary dynamics is shaped by social learning[10,46,47], whereby the most successful individuals will tend to be imitated more often by the others. In the current work, social learning is modeled using the so-called pairwise comparison rule[48], assuming that an individual A with fitness f_A adopts the strategy of another individual B with fitness f_B with probability given by the Fermi function, $\left(1 + e^{-\beta(f_B - f_A)}\right)^{-1}$. The parameter β represents the 'imitation strength' or 'intensity of selection', i.e., how strongly the individuals base their decision to imitate on fitness comparison. For $\beta = 0$, we obtain the limit of neutral drift – the imitation decision is random. For large β, imitation becomes increasingly deterministic.

In the absence of mutations or exploration, the end states of evolution are inevitably monomorphic: once such a state is reached, it cannot be escaped through imitation. We thus further assume that, with a certain mutation probability, an individual switches randomly to a different strategy without imitating another individual. In the limit of small mutation rates, the behavioral dynamics can be conveniently described by a Markov Chain, where each state represents a monomorphic population, whereas the transition probabilities are given by the fixation probability of a single mutant[9,45,49]. The resulting Markov Chain has a stationary distribution, which characterizes the average time the population spends in each of these monomorphic end states.

In finite populations, the groups engaging in PGG are given by multivariate hypergeometric sampling. For transition between two pure states (small mutation rate), this reduces to sampling (without replacement) from a hypergeometric distribution[9]. Denote

$$H(k, N, m, Z) = \frac{\binom{m}{k}\binom{Z-m}{N-k}}{\binom{Z}{N}}$$

Let $\Pi_{ij}(k)$ and $\Pi_{ji}(k)$ denote the payoff of a strategists of type i and j, respectively, when the random sampling consists of k players of type i and $N - k$ players of type j (as given in the payoff matrix in Equation (1) in SI). Hence, in a population of x i-strategists and $(Z-x)$ j-strategists, the average payoffs to $i-$ and $j-$ strategists are[9,10]:

$$
\begin{aligned}
Pij(x) &= \sum_{k=0}^{N-1} H(k, N-1, x-1, Z-1)\, \Pi_{ij}(k+1) \\
&= \sum_{k=0}^{N-1} \frac{\binom{x-1}{k}\binom{Z-x}{N-1-k}}{\binom{Z-1}{N-1}}\, \Pi_{ij}(k+1)
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
Pji(x) &= \sum_{k=0}^{N-1} H(k, N-1, x, Z-1)\, \Pi_{ji}(k) \\
&= \sum_{k=0}^{N-1} \frac{\binom{x}{k}\binom{Z-1-x}{N-1-k}}{\binom{Z-1}{N-1}}\, \Pi_{ji}(k)
\end{aligned}
\tag{13}
$$

Note that several $P_{ij}(x)$ can be further simplified (see Supporting Information). Now, the probability to change the number k of individuals using strategy i by ± 1 in each time step can be written as[48]

$$
T^{\pm}(k) = \frac{Z-k}{Z}\frac{k}{Z}\left[1 + e^{\mp\beta[P_{ij}(k) - P_{ji}(k)]}\right]^{-1}. \tag{14}
$$

The fixation probability of a single mutant with a strategy i in a population of $(Z-1)$ individuals using j is given by[45,48–50]

$$
\rho_{j,i} = \left(1 + \sum_{i=1}^{N-1} \prod_{j=1}^{i} \frac{T^{-}(j)}{T^{+}(j)}\right)^{-1}. \tag{15}
$$

In the limit of neutral selection (i.e. $\beta = 0$), $\rho_{B,A}$ equals the inverse of population size, $1/Z$.

Considering a set $\{1, ..., q\}$ of different strategies, these fixation probabilities determine a transition matrix $M = \{T_{ij}\}_{i,j=1}^{q}$, with $T_{ij,j\neq i} = \rho_{ji}/(q-1)$ and $T_{ii} = 1 - \sum_{j=1,j\neq i}^{q} T_{ij}$, of a

Markov Chain. The normalized eigenvector associated with the eigenvalue 1 of the transposed of M provides the stationary distribution described above[45,49,50], describing the relative time the population spends adopting each of the strategies.

4.2 Risk-dominance condition

An important analytical criteria to determine the viability of a given strategy is whether it is risk-dominant with respect to other strategies[13,32]. Namely, one considers which selection direction is more probable: an i mutant fixating in a homogeneous population of individuals playing j or a j mutant fixating in a homogeneous population of individuals playing i. When the first is more likely than the latter, i is said to be *risk-dominant* against j[32], which holds for any intensity of selection and in the limit of large Z when

$$\sum_{k=1}^{N} \Pi_{ij}(k) \geq \sum_{k=0}^{N-1} \Pi_{ji}(k). \tag{16}$$

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Figure Legends

Figure 1. (a) Stationary distribution and fixation probabilities. The population spends most of the time in the homogenous state of AVOID. The black arrows identify the advantageous transitions, where $\rho_N = 1/Z$ denotes the neutral fixation probability. The dashed lines denote neutral transitions. Note the cyclic pattern from cooperation to defection to commitment strategies and back. **(b) Contour plot of the frequency of AVOID as a function of ϵ_P and δ**. For a small enough cost of arranging the commitment, AVOID is abundant whenever a sufficient compensation is associated with the commitment deal. Parameters: $N = 5$, $Z = 100$, $r = 3$; $\beta = 0.1$; In panel a, $\epsilon_P = 0.25,\ \delta = 2$.

Figure 2. (a) Transition probabilities and stationary distributions in case of RESTRICT. For an efficient restriction ($\epsilon_R = 0.5$ and $\psi = 0.25$), the population spends most of the time in the homogenous state of RESTRICT. Notations are the same as in Figure 1a. **(b) Frequencies of each strategy for varying ψ, in case of RESTRICT**. For a given cost of restriction ($\epsilon_R = 0.5$), in general the better the effect of restriction on non-committers (i.e. the smaller ψ), the greater the frequency of RESTRICT. **(c) Frequency of RESTRICT as a function of ϵ_R and ψ, in a population with C, D, FREE and FAKE strategies.** For a large range of cost for restricting the access of non-committers, ϵ_R, and the restriction, ψ, RESTRICT is highly frequent, having a higher frequency than AVOID. The double-stroke line corresponds to the part having the same frequency as AVOID (i.e. 0.64, with the same parameter values), and the area below this line identifies the area in which RESTRICT is more frequent than AVOID. In general, the larger ϵ_R, the smaller ψ is required for RESTRICT to be advantageous to AVOID. **(d) Frequencies of each strategy as a function of the group size, N.** RESTRICT becomes more frequent when the group size increases, even for a rather high cost of restriction ($\epsilon_R = 2.0$). Parameters: In panels a, b, c: $N = 5$; in all cases, $Z = 100$, $r = 3$; $\epsilon_P = 0.25, \delta = 2$; $\beta = 0.1$.

Figure 3. (a) Range of parameters ψ, ϵ_R and ϵ_P, generated from the analytical formula in Equation (10), in which RESTRICT is better than AVOID. For a large range of cost for restricting the access of non-committers, ϵ_R, and the effect of restriction, ψ, RESTRICT is better than AVOID. In general, the larger ϵ_R, the smaller ψ is required for RESTRICT to be advantageous to AVOID. **(b) Group size is an important factor for making RESTRICT more viable than AVOID.** We compute, as a function of the group size, N, the frequencies of RESTRICT for different values of restriction cost ϵ_R (the curves without markers), in comparison to the frequency of AVOID (the red curve with circled markers). In general, the lower the cost of restriction, the higher the frequency of RESTRICT. Also, the threshold of N above which RESTRICT is more frequent than AVOID is smaller. Parameters: in panel b, $Z = 100$, $\epsilon_P = 0.25$, $\psi = 0.25$, $\beta = 0.1$; In both panels, $N = 5$, $r = 3$.

Supporting Information: Avoiding or Restricting Defectors in Public Goods Games?

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1 Payoff formulas

First, we derive the the payoffs $\Pi_{ij}(k)$ for the five strategies AVOID (or RESTRICT), C, FREE, D, and FAKE (denoted 1, 2, 3, 4, 5, respectively, as in the main text). Recall that $\Pi_{ij}(k)$ denotes the payoff of a strategist of type i (resp., type j) when the random sampling consists of k players of type i and $N-k$ players of type j.

Denote $\Pi(k) = \{\Pi_{ij}(k)\}_{i,j=1,i\neq j}^{5}$, where, abusing notation, k is the number of AVOID (or RESTRICT) players if they are present in the pair; otherwise, the number of C players if C is present in the pair. Except for $\Pi_{31}(0) = \Pi_{51}(0) = 0$, we have

$$\Pi(k) = \begin{array}{c} \\ 1 \\ C \\ FREE \\ D \\ FAKE \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & C & FREE & D & FAKE \end{array} \\ \begin{pmatrix} - & rc-c-\frac{\epsilon_P}{k} & rc-c-\frac{\epsilon_P}{k} & \Pi_{14}(k) & (\frac{rk}{N}-1)c+\frac{N\delta-\epsilon_P}{k}-\delta \\ rc-c & - & \frac{rk}{N}c-c & \frac{rk}{N}c-c & \frac{rk}{N}c-c \\ rc-c & \frac{rk}{N}c & - & 0 & 0 \\ \Pi_{41}(k) & \frac{rk}{N}c & 0 & - & 0 \\ \frac{rk}{N}c-\delta & \frac{rk}{N}c & 0 & 0 & - \end{pmatrix} \end{array} \tag{1}$$

where

- for RESTRICT, $\Pi_{14}(k) = \frac{rkc}{k+\psi(N-k)} - c - \frac{\epsilon_P+\epsilon_R}{k}\ \forall 1 \leq k \leq N$ and $\Pi_{41}(k) = \frac{rkc\psi}{k+\psi(N-k)}\ \forall 1 \leq k \leq N-1$ and $\Pi_{41}(0) = 0$;
- for AVOID, $\Pi_{14}(N) = rc - c - \frac{\epsilon_P}{N}$ and $\Pi_{14}(k) = 0\ \forall 1 \leq k \leq N-1$, and $\Pi_{41}(k) = 0\ \forall 0 \leq k \leq N-1$.

We now derive the average payoffs $Pij(x)$ and $Pji(x)$ defined in the main text. For simplicity, consider $c = 1$. We have

$$P_{12}(x) = P_{13} = \sum_{k=0}^{N-1} H(k, N-1, x-1, Z-1)\, (r - 1 - \frac{\epsilon}{k+1}) = r - 1 - \frac{\binom{Z}{N} - \binom{Z-x}{N}}{x\binom{Z-1}{N-1}}\epsilon_P$$

$$P_{21}(x) = \sum_{k=0}^{N-1} H(k, N-1, x, Z-1)\, (r-1) = r - 1$$

$$P_{31}(x) = \sum_{k=1}^{N-1} H(k, N-1, x, Z-1)\, (r-1) = (r-1)\left(1 - \frac{\binom{Z-1-x}{N-1}}{\binom{Z-1}{N-1}}\right)$$

$$P_{15}(x) = \sum_{k=0}^{N-1} H(k, N-1, x-1, Z-1)\, (\frac{r(k+1)}{N} + \frac{N\delta - \epsilon_P}{k+1} - \delta - 1) =$$

$$= \frac{r}{N}\left(1 + (x-1)\frac{N-1}{Z-1}\right) + \frac{\binom{Z}{N} - \binom{Z-x}{N}}{x\binom{Z-1}{N-1}}(N\delta - \epsilon_P) - \delta - 1$$

$$P_{51}(x) = \sum_{k=1}^{N-1} H(k, N-1, x-1, Z-1)\, (\frac{rk}{N} - \delta) = \frac{r(N-1)}{N(Z-1)}x - \delta\left(1 - \frac{\binom{Z-1-x}{N-1}}{\binom{Z-1}{N-1}}\right)$$

$$P_{23}(x) = P_{24} = P_{25} = \frac{r}{N}\left(1 + (x-1)\frac{N-1}{Z-1}\right) - 1$$

$$P_{32}(x) = P_{42} = P_{52} = \frac{r(N-1)}{N(Z-1)}x$$

$$P_{34}(x) = P_{43} = P_{35} = P_{53} = P_{45} = P_{54} = 0$$

For AVOID: $P_{41}(x) = 0$ and $P_{14}(x) = \dfrac{\binom{x-1}{N-1}}{\binom{Z-1}{N-1}}(r - 1 - \frac{\epsilon_P}{N})$

For RESTRICT: P_{14} and P_{41} are hard to compute analytically, we follow the sum formulas in our numerical simulations.

2 Some simplifications of the analytical results and proofs

2.1 Some simplifications

Here, using the well-known inequalities[4]

$$\log N + \gamma < F_N = \sum_{k=1}^{N} \frac{1}{k} \leq \log N + 1$$

where $\gamma = 0.577215$, we provide some simplifications of the conditions obtained in the main text. First of all, regarding the conditions for risk-dominance of AVOID against D, FREE and FAKE:

$$\begin{aligned} \epsilon_P &\leq \frac{c(r-1)}{\log N + \gamma} \\ \delta &\geq \frac{N-r}{NF_{N-1}} c + \frac{F_N}{NF_{N-1}} \epsilon_P. \end{aligned} \tag{2}$$

They can be simplified to

$$\begin{aligned} \epsilon_P &\leq c(r-1)/F_N \\ \delta &\geq \frac{(N^2 - rN)c + \epsilon_P}{N^2 \left(\log(N-1) + 1\right)} + \frac{\epsilon_P}{N}. \end{aligned} \tag{3}$$

Now, the necessary condition for RESTRICT to be risk-dominant against D, which is

$$\epsilon_P + \epsilon_R \leq \frac{N(r-1)}{F_N} c, \tag{4}$$

can be simplified to

$$\epsilon_P + \epsilon_R < \frac{N(r-1)}{\log N + \gamma} c. \tag{5}$$

Furthermore, the necessary condition for RESTRICT to be favored to AVOID

$$(r-1)c \geq \frac{F_N}{N-1} \epsilon_R + \frac{F_{N-1}}{N-1} \epsilon_P \tag{6}$$

can be simplified to

$$(r-1)c \geq \frac{\epsilon_R(\log N + \gamma) + \epsilon_P(\log(N-1) + \gamma)}{N-1} \tag{7}$$

2.2 Some proofs

2.2.1 Ratio of fixation probabilities

It has been shown that[5]

$$\frac{\rho_{j,i}}{\rho_{i,j}} = \prod_{k=1}^{N-1} \frac{T^-(k)}{T^+(k)} = \prod_{k=1}^{N-1} \frac{1+e^{\beta[P_{ij}(k)-P_{ji}(k)]}}{1+e^{-\beta[P_{ij}(k)-P_{ji}(k)]}} = e^{\beta \sum_{k=1}^{N-1}(P_{ij}(k)-P_{ji}(k))}$$

Hence, considering two different strategies j and j', the inequality

$$\frac{\rho_{j,i}}{\rho_{i,j}} \geq \frac{\rho_{j',i}}{\rho_{i,j'}}$$

holds if and only if

$$\sum_{k=1}^{N-1} (\pi_{ij}(k) - P_{ji}(k)) \geq \sum_{k=1}^{N-1} (P_{ij'}(k) - P_{j'i}(k))$$

This can be further simplified, in large population limit, to[1]

$$\sum_{k=1}^{N} P_{ij}(k) - \sum_{k=0}^{N-1} P_{ji}(k) \geq \sum_{k=1}^{N} P_{ij'}(k) - \sum_{k=0}^{N-1} P_{j'i}(k)$$

2.2.2 Decrease of F_N/N and $F_N/(N-1)$

We prove that $F_N/N > F_{N+1}/(N+1)$ and that $F_N/(N-1) > F_{N+1}/N$. Indeed, we have

$$\begin{aligned} (N+1)F_N - NF_{N+1} &= N(F_N - F_{N+1}) + F_N = F_N - \frac{N}{N+1} \\ &= \sum_{k=1}^{N}(\frac{1}{k} - \frac{1}{N+1}) > 0 \end{aligned} \tag{8}$$

Moreover,

$$NF_N - (N-1)F_{N+1} = N(F_N - F_{N+1}) + F_{N+1} > N(F_N - F_{N+1}) + F_N > 0 \tag{9}$$

Furthermore, since $\lim_{N\to+\infty} F_N = \log N + \gamma$[4], we have

$$\lim_{N\to+\infty} \left(\frac{F_N}{N-1}\epsilon_R + \frac{F_{N-1}}{N-1}\epsilon_P \right) = 0.$$

2.2.3 Properties of the function in Equation (10) in the main text

Consider the following formula

$$\sum_{k=1}^{N-1} \frac{k(1-\psi)}{\psi N + k(1-\psi)} rc - F_N \epsilon_R - F_{N-1}\epsilon_P - (N-1)c. \tag{10}$$

It is clear that it is a decreasing function of ϵ_R and ϵ_P since F_N and F_{N-1} are positive. It increases with r for a similar reason. Moreover, it decreases with ψ since

$$\frac{k(1-\psi)}{\psi N + k(1-\psi)} = \frac{k}{k + N\frac{\psi}{1-\psi}} = \frac{k}{k + N\left(\frac{1}{1-\psi} - 1\right)} \tag{11}$$

is a decreasing function of $\psi \in (0,1)$ for all $1 \leq k \leq N-1$.

Furthermore, when r tends to infinity, fixing other parameters, ψ (and hence also its threshold below which RESTRICT is better than AVOID, ψ^{AVOID}) tends to 1 since

$$\lim_{r \to +\infty} \frac{F_N \epsilon_R - F_{N-1}\epsilon_P - (N-1)c}{rc} = 0$$

and

$$\sum_{k=1}^{N-1} \frac{k(1-\psi)}{\psi N + k(1-\psi)} = 0 \text{ at } \psi = 1.$$

3 Performance of AVOID and RESTRICT depending on the arrangement cost

In Fig. S1 we show the frequencies of the five strategies in case of AVOID and RESTRICT for varying the cost of arranging commitment ϵ_P. In general, the smaller this cost, the higher the frequency of AVOID and RESTRICT. For small cost of arranging commitment, both AVOID and RESTRICT are highly frequent, dominating their population. When the cost is sufficiently large, in case of AVOID the commitment free-riders FREE takes over. This observation is similar to the pairwise case[2]. But in case of in case of RESTRICT the non-committers D take over. Note that AVOID players do not have to pay this cost when playing with D because no game is played between these strategies (see the models in the main text), while RESTRICT

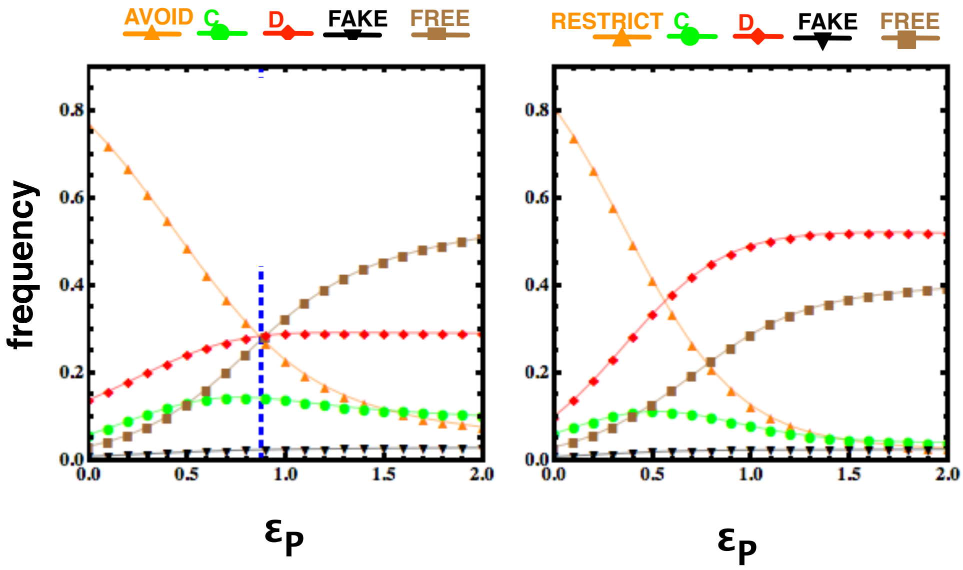

Figure S1: Frequency of each strategy in case of AVOID (left) and RESTRICT (right) for varying ϵ_P. For small cost of arranging commitment, both AVOID and RESTRICT are dominant, while commitment free-riders FREE takes over when the cost is high in the first case, and the non-committers take over in the second case. The blue line is the analytical threshold (derived in the main text of ϵ_P) for which AVOID is risk-dominant against all defectors and free-riders. Clearly, analytical results complies with numerical ones. Parameters: In the right panel, $\epsilon_R = 1.0$; In both cases, $N = 5$, $Z = 100$, $r = 3$, $\delta = 2$; $\beta = 0.1$;

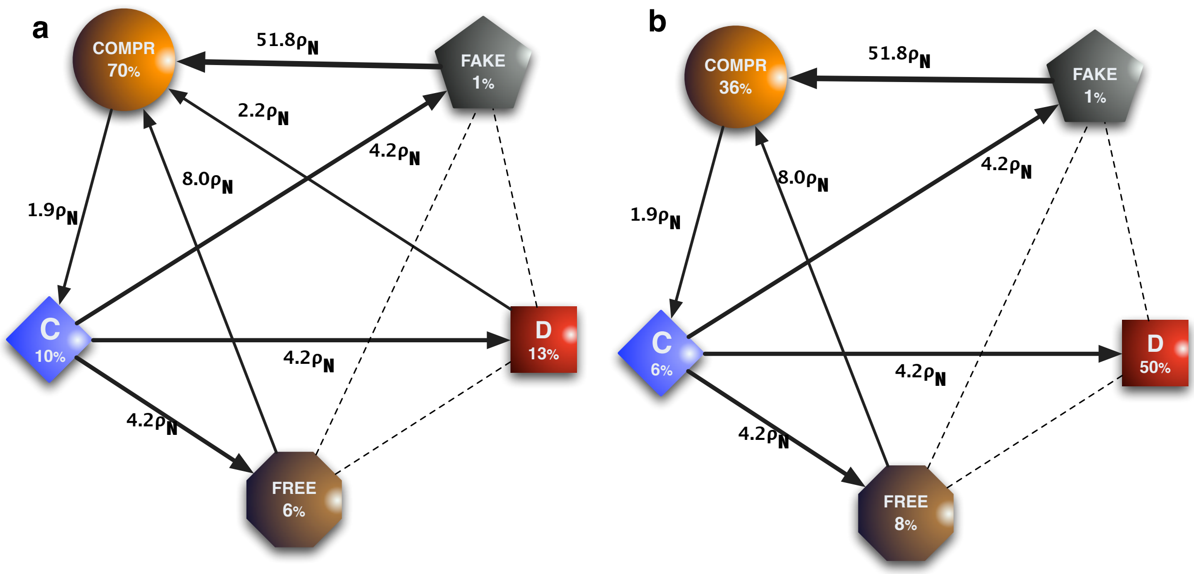

Figure S2: Transition probabilities and stationary distributions in case of RESTRICT. For a given cost of restriction ϵ_R, the better the effect of restriction on non-commiters D, the better RESTRICT. Note the arrow from D to RESTRICT for small ψ **(panel a,** $\psi = 0.25$) which disappears when ψ is large **(panel b,** $\psi = 0.5$). Parameters: $N = 5$, $Z = 100$, $r = 3$; $\epsilon_R = 0.5$; $\beta = 0.1$;

players have to (and also the cost of restriction ϵ_R) when playing with D. We therefore see additionally that in case of AVOID when ϵ_P is sufficiently large, D does not increase in terms of frequency while it does so in case of RESTRICT.

4 Contour plots for AVOID with varying N

For varying N, AVOID is abundant whenever a sufficient compensation is associated with the commitment deal, see Figure S3. Hence, ϵ_P is the essential parameter deciding whether the commitment strategy is successful. Furthermore, when the cost is small the frequency of AVOID decreases with group size; but when the cost is sufficiently large this frequency increases. It is like when we have a good law-enforcing system which reduces the cost of arranging commitment: then AVOID can lead to better cooperation; but once that cost cannot be reduced sufficiently, then interacting in larger groups is actually better for AVOID because the cost is shared between more AVOID players.

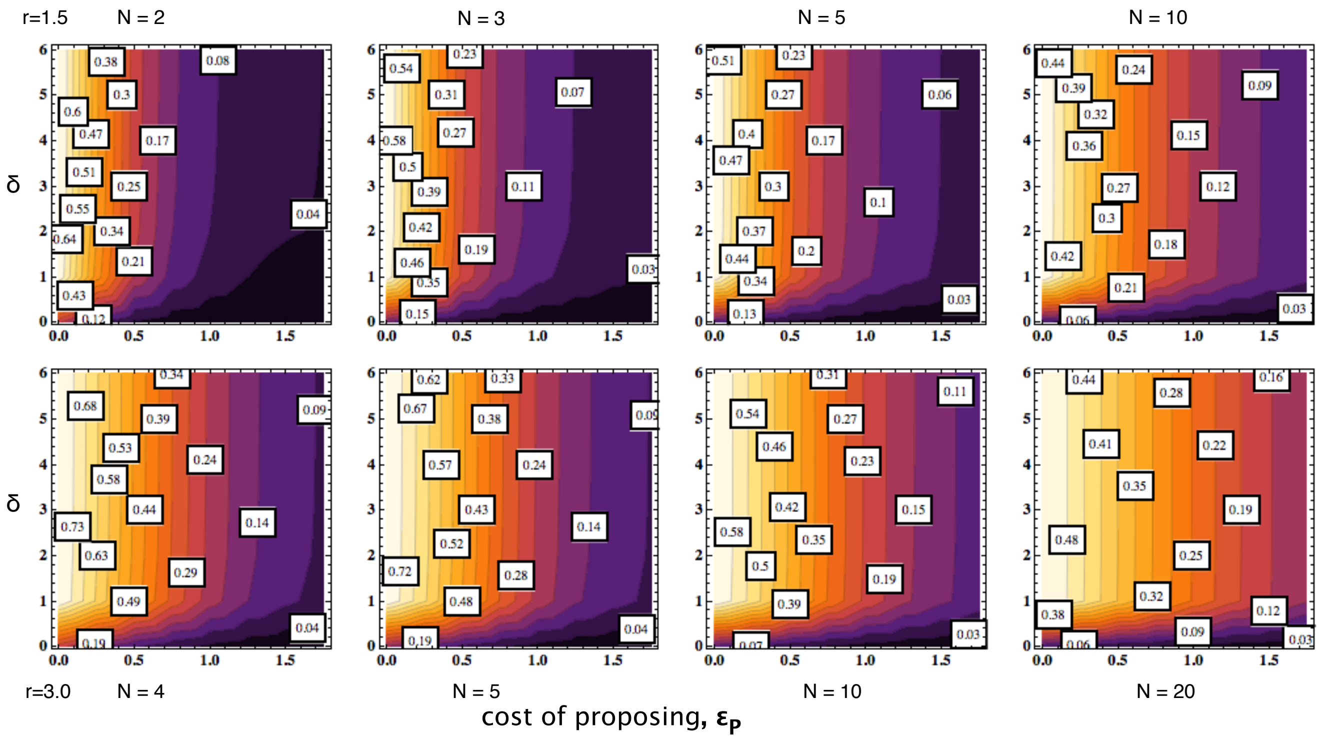

Figure S3: Contour plot of the frequency of AVOID as a function of ϵ_P and δ, for different group sizes N. Parameters: $Z = 100$, $\beta = 0.1$. In general, for small enough cost of arranging the commitment, AVOID is abundant whenever a sufficient compensation is associated with the commitment deal. That is, ϵ_P is the essential parameter for the commitment strategy. Nonetheless, for small ϵ_P the frequency of AVOID decreases with N, while for larger ϵ_p, it increases.

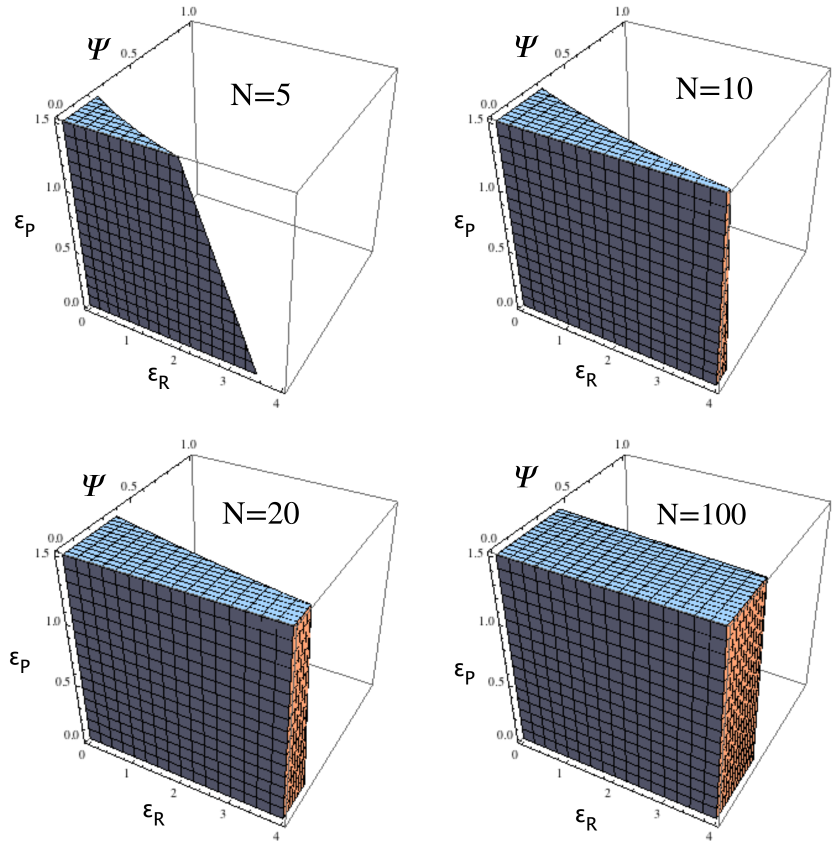

Figure S4: Range of parameters ψ, ϵ_R and ϵ_P, generated from the analytical formula in Eq. (10) in the main text, in which RESTRICT is better than AVOID, for different values of N. In general, the larger N, the larger the parameter space in which RESTRICT is advantageous to AVOID in dealing with non-committers D. Parameters: $Z = 100$, $\epsilon_P = 0.25$, $r = 3$.

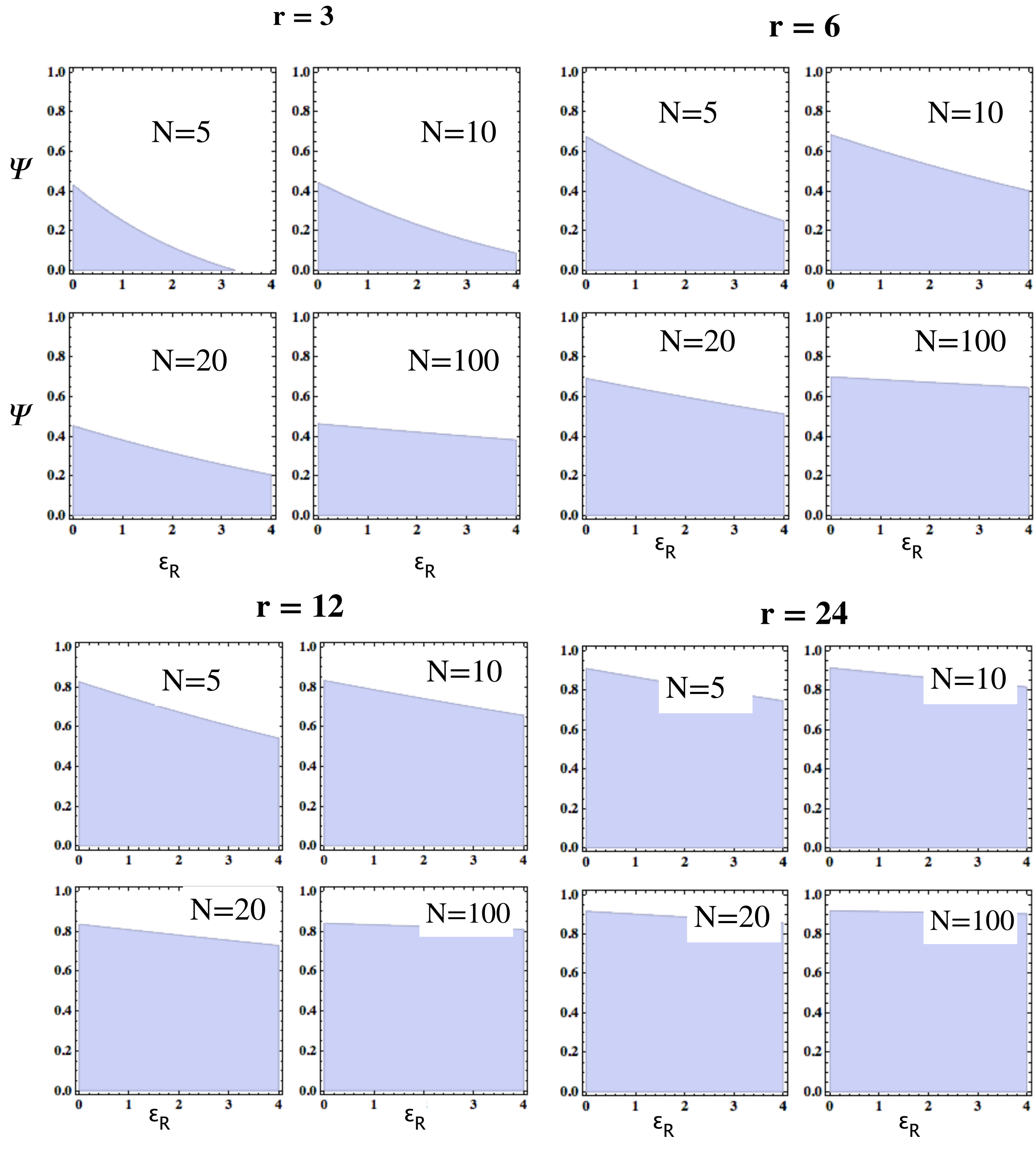

Figure S5: Range of parameters ψ, ϵ_R and ϵ_P, generated from the analytical formula in Eq. ... in the main text, in which RESTRICT is better than AVOID, for different values of N and r. In general, the larger r and N, the larger the parameter space in which RESTRICT is advantageous to AVOID in dealing with non-committers D. Parameters: $Z = 100$, $\epsilon_P = 0.25$.

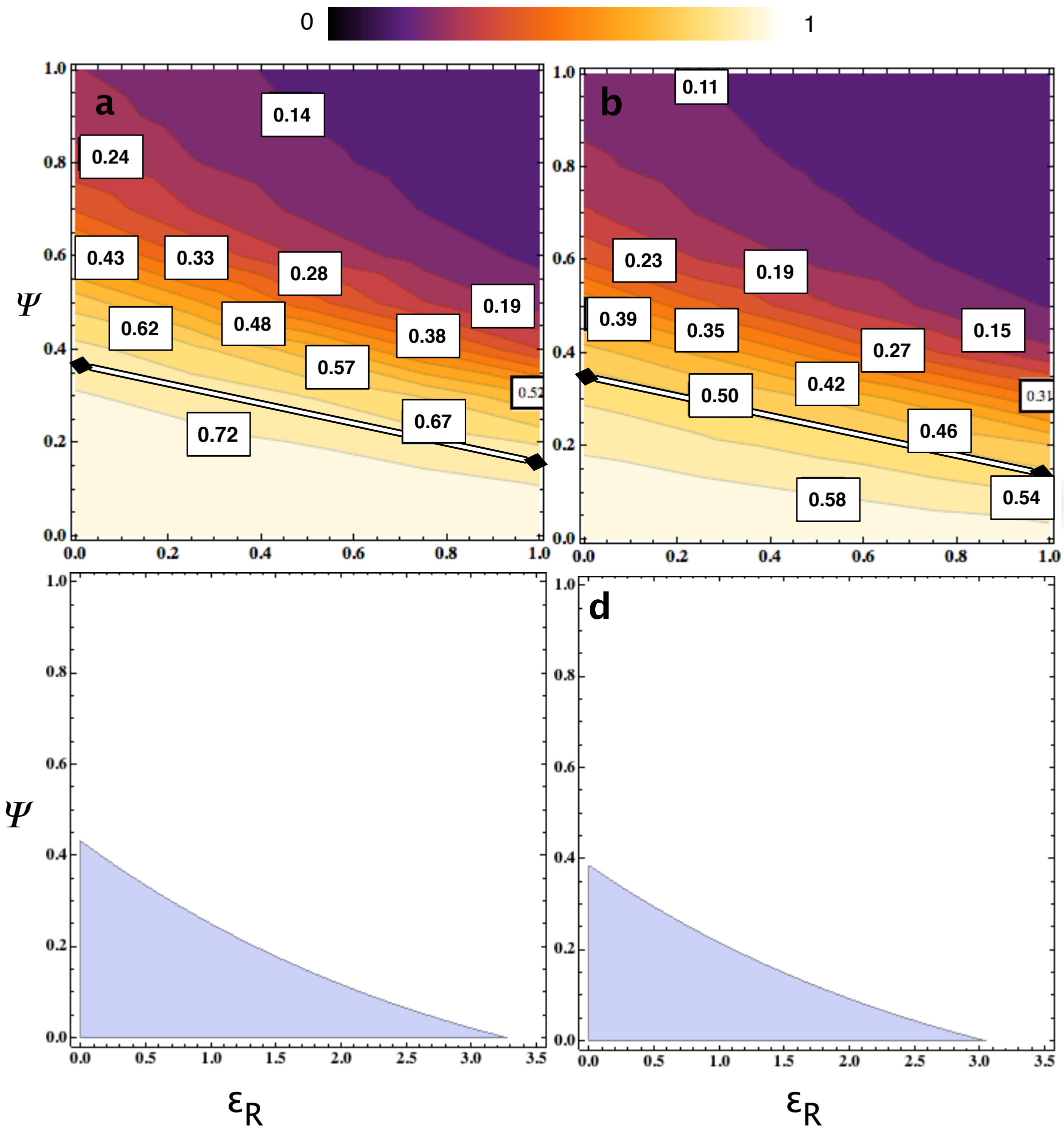

Figure S6: Frequency of RESTRICT as a function of ϵ_R and ψ, with (a) $\epsilon_P = 0.25$ and (b) $\epsilon_P = 0.5$. For a large range of cost for restricting the access of non-committers, ϵ_R, and the restriction, ψ, RESTRICT is better than AVOID. See the area below the double-stroke curves, which corresponds to the frequency of AVOID (0.64 in panel a and 0.49 in panel b). In general, the larger ϵ_R, the smaller ψ required for RESTRICT to be advantageous to AVOID. This clearly complies with **analytical results generated by Eq. (10) in the main text, as shown in the panels (c) $\epsilon_P = 0.25$ and (d) $\epsilon_P = 0.5$.** Interestingly, ψ is the decisive parameter on the frequency of RESTRICT. Parameters: $N = 5$, $Z = 100$, $r = 3$; $\delta = 2$; $\beta = 0.1$.

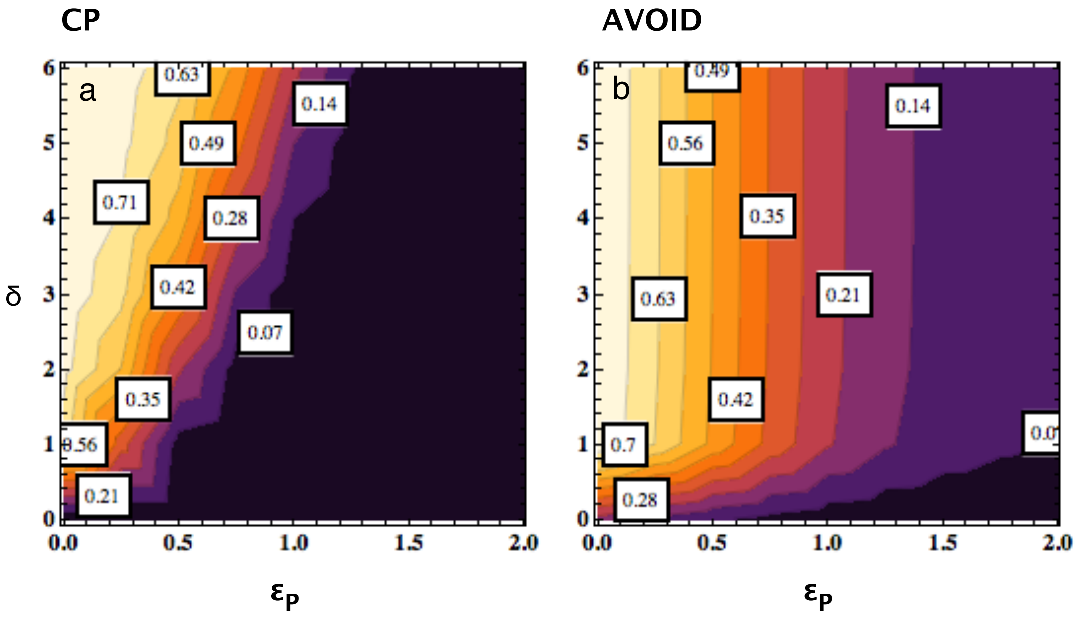

Figure S7: Costly peer punishment (CP) versus AVOID. (a) Fraction of CP in a population with C and D; (b) fraction of AVOID in a population with C, D, FREE and FAKE. Parameters: $N = 5$, $Z = 100$, $r = 3$; $\delta = 2$; $\epsilon_P = 0.25$; $\beta = 0.1$.

5 RESTRICT vs. AVOID for varying N and r

We generate analytical results using Eq. (10) in the main text, describing the parameter space where RESTRICT is better than AVOID in dealing with non-committers (hence, becomes more frequent in the population with the other four non-proposing strategies). In general, the larger N, the larger the parameter space in which RESTRICT is advantageous to AVOID in dealing with non-committers, see Figure S4.

In Fig. S5 we show similar results for varying the public goods producing factor r. The results show that the larger r, the larger parameter space where RESTRICT is advantageous to AVOID. It complies with the Eq (10) in the main text, the left hand size of which is clearly an increasing function of r.

In Fig. S6 we also show that these analytical results corroborate with the the numerical simulations.

6 Simple Punishment vs. AVOID

A costly peer punishment strategy, CP, in the PGG game, contributes to the public good. After the PGG was played, the punisher can impose a fine δ upon each non-contributor (defector) D, at a personal cost ϵ_P (see more details in reference[3]).

Figure S7 shows that, differently from AVOID where ϵ is the crucial parameter as long as δ is sufficiently large, the frequency of CP always increases with δ. We observe that AVOID is more frequent than CP most of the time.

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