A Well-founded semantics for hybrid MKNF knowledge bases

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1 Introduction

In recent year increasing interest has been witnessed in the topic of integrating description logics (DL) and (nonmonotonic) logic programming(LP). Several approaches have appeared that deal with hybrid knowledge bases that include one part described as a description logic theory, and another as a logic program with negation by default (e.g. [10, 2, 3, 5, 8]).

Much of this research effort is being driven by the needs of the Semantic Web initiative. Indeed, the addition of rules on top of the DL-based ontology layer has been recognized as an important task for the success of the Semantic Web, and initiatives are being taken to define such a rule layer (cf. the Rule Interchange Format working group of the W3C). The combination in the Semantic Web of rules, in LP style, and ontologies, described using DL, is not trivial since these two formalisms are based on fundamental different assumptions: the former is a nonmonotonic formalism, relying on some form of closed world assumption, while the latter is based on first order logics, and uses open world assumption.

As mentioned above, various approaches for the combination of LP and DL formalisms already exist. But with the exception of [3], those approaches rely on the stable models semantics (SMS) of logic programs [4]. It is our stance that, especially for use in the Semantic Web, another choice of basic semantics for the logic programming part is needed, more precisely an approach based on the well-founded semantics (WFS) [12]. Compared to the stable models semantics, the well founded semantics of logic programs has several features that we believe make it more amenable for the needs of the Semantic Web. In applications that require the capability of dealing with an overwhelming mass of information, it is very important to be able to quickly process such information, even at the cost of losing some inference power. In this respect, as it is well known, the computation of stable models of LPs is NP-hard, whereas that of the WFS is polynomial. Another requirement not fulfilled by SMS is that of being able to answer queries about a given part of the program without the need to, in general, consult the whole program. On the contrary, with the WFS it is possible to implement query driven proof procedures that, for any given query, only need to explore a part of the program. Moreover, the WFS is defined for all programs, while the SMS may fail to assign a meaning to some (consistent) programs. These arguments for the choice of WFS as the basis on which the meaning of combination of DL with LP is defined are also acknowledge by [3]. However, the combination proposed by [3] relies on a loose coupling with several limitations. These limitations are not present in approaches where the DL and the LP part are more tightly integrated[3]. Among these approaches we highlight the approach of hybrid MKNF knowledge bases of [8] which is built on top of the logic of Minimal Knowledge and Negation as failure (MKNF) of [6].

With the motivation above, in this paper we define a well founded semantics for hybrid MKNF knowledge bases, for now restricting to nondisjunctive MKNF rules. Our proposal compares to that of [8] along the same lines as the SMS compares to WFS of LP: our well founded semantics is a sound approximation of the semantics of [8]; the computational complexity is strictly lower; when the DL knowledge base is consistent the existence of well founded models is guaranteed. Moreover, the semantics retains the property of [8] of being faithful, but now w.r.t. the WFS. More precisely, when the DL part is empty, the semantics exactly coincides with the WFS of LPs.

After this introduction, we start the paper by recalling some basic notion of the MKNF logics, and of hybrid MKNF knowledge bases. Then, in order to be able to define the well founded semantics which relies on 3-valued logics, we generalize the notion of model to this 3-valued setting. The paper continues with the definition of the proposed semantics for hybrid KBs, and some of its properties. We end by drawing some conclusion and presenting future work. Lack of space prevents us from presenting here all proofs, which can be found in the extended report at http://centria.di.fct.unl.pt/ mknorr/wfmknf-extd.pdf

2 Preliminaries

MKNF notions. We start by defining the syntax of MKNF formulas from [7]. A *first-order atom* $p(t_1, \ldots, t_n)$ is an MKNF formula where p is a predicate and the t_i are first-order terms[4]. If φ is an MKNF formula then $\neg\varphi$, $\exists x : \varphi$, $\mathbf{K}\,\varphi$ and $\mathbf{not}\,\varphi$ are MKNF formulas and likewise $\varphi_1 \wedge \varphi_2$ and $\varphi_1 \subset \varphi_2$ for MKNF formulas φ_1, φ_2. We use the following well-known symbols to represent particular boolean combinations of the previously introduced syntax, i.e. $\varphi_1 \vee \varphi_2$ for $\neg(\neg\varphi_1 \wedge \neg\varphi_2)$, $\varphi_1 \equiv \varphi_2$ for $(\varphi_1 \subset \varphi_2) \wedge (\varphi_2 \subset \varphi_1)$, and $\forall x : \varphi$ for $\neg\exists x : \neg\varphi$. Substituting the free variables x_i in φ by terms t_i is denoted $\varphi[t_1/x_1, \ldots, t_n/x_n]$. Given a (first-order) formula φ, $\mathbf{K}\,\varphi$ is called a *modal* **K**-*atom* and $\mathbf{not}\,\varphi$ a *modal* **not**-*atom* and both are *modal atoms*. If a modal atom does not occur in scope of a modal operator in an MKNF formula then it is *strict*. An MKNF formula φ without any free variables is a *sentence* and *ground* if it does not contain variables at all. It is *modally closed* if all modal operators (**K** and **not**) are applied in φ only to sentences and φ is *positive* if it does not contain the operator **not**. Moreover, φ is *subjective* if all first-order atoms of φ occur within the scope of a modal

[3] For a comparison between loose coupling coupling and hybrid MKNF knowledge bases see e.g. [7].

[4] We consider function-free first-order logic, so the terms are either constants or variables.

operator. Finally, φ is *flat* if φ is subjective and all occurrences of modal atoms in φ are strict.

Recalling [7], *standard names assumption* is employed, i.e. apart from the constants occurring in the formulas the signature contains a countably infinite supply of constants not occurring in the formulas. The Herbrand Universe of such a signature is also called *domain set* $\triangle$. An *MKNF structure* is a triple (I, M, N) where I is an Herbrand first-order interpretation over $\triangle$ and M and N are nonempty sets of Herbrand first-order interpretations over $\triangle$ and the equality predicate $\approx$ is interpreted as congruence relation in I and in each interpretation occurring in M, N. For the 2-valued satisfiability of MKNF sentences we refer to [7], all the more so since in the next section we define 3-valued satisfiability in a way that, when restricted to 2-values trivially coincides with the one of [7].

Hybrid MKNF Knowledge Bases. Quoting from [8], the approach of hybrid MKNF knowledge bases is applicable to any first-order fragment $\mathcal{DL}$ satisfying these conditions: (i) each knowledge base $\mathcal{O} \in \mathcal{DL}$ can be translated into a formula $\pi(\mathcal{O})$ of function-free first-order logic with equality, (ii) it supports *A-Boxes*-assertions of the form $P(a_1, \ldots, a_n)$ for P a predicate and a_i constants of $\mathcal{DL}$ and (iii) satisfiability checking and instance checking (i.e. checking entailments of the form $\mathcal{O} \models P(a_1, \ldots, a_n)$ are decidable[5].

We recall MKNF rules and hybrid MKNF knowledge bases from [8].

Definition 2.1. *Let $\mathcal{O}$ be a DL knowledge base. A first-order function-free atom $p(t_1, \ldots, t_n)$ over Σ such that p is $\approx$ or it occurs in $\mathcal{O}$ is called a* DL-atom*; all other atoms are called* non-DL-atoms*. An MKNF rule r has the following form where H_i, A_i, and B_i are first-order function free atoms:*

$$\mathbf{K}\, H_1 \vee \ldots \vee \mathbf{K}\, H_l \leftarrow \mathbf{K}\, A_1, \ldots, \mathbf{K}\, A_n, \mathbf{not}\, B_1, \ldots, \mathbf{not}\, B_m \tag{1}$$

The sets $\{\mathbf{K}\, H_i\}$, $\{\mathbf{K}\, A_i\}$, and $\{\mathbf{not}\, B_i\}$ are called the rule head*, the* positive body*, and the* negative body*, respectively. A rule is* nondisjunctive *if $l = 1$; r is* positive *if $m = 0$; r is a* fact *if $n = m = 0$; r is* safe *if all variables in r occur in a positive body atom. A* program *is a finite set of MKNF rules. A* hybrid MKNF knowledge base *$\mathcal{K}$ is a pair $(\mathcal{O}, \mathcal{P})$ and $\mathcal{K}$ is* nondisjunctive *if all rules in $\mathcal{P}$ are nondisjunctive.*

The semantics of an MKNF knowledge base is obtained by transforming it into an MKNF formula.

Definition 2.2. *Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a hybrid MKNF knowledge base, and let $\pi(\mathcal{O})$ be the translation of $\mathcal{O}$ into first-order logic with equality. For an MKNF rule r, let x be the vector of variables occurring in the rule. We extend π to r, $\mathcal{P}$, and $\mathcal{K}$ as follows:*

$$\begin{aligned} \pi(r) &= \forall x : (H_1 \vee \ldots \vee H_n \subset B_1, \ldots, B_m) \\ \pi(\mathcal{P}) &= \textstyle\bigwedge_{r \in \mathcal{P}} \pi(r) \\ \pi(\mathcal{K}) &= \mathbf{K}\, \pi(\mathcal{O}) \wedge \pi(\mathcal{P}) \end{aligned}$$

[5] For more details on DL notation we refer to [1].

For the rationales behind these notions see [7] and [8]. We also recall the notion of DL-safeness.

Definition 2.3. *([7]) An MKNF rule r is* DL-safe *if every variable in r occurs in at least one non-DL-atom $\mathbf{K}\, B$ occurring in the body of r. A hybrid MKNF knowledge base $\mathcal{K}$ is* DL-safe *if all its rules are DL-safe.*

Given a hybrid MKNF knowledge base $\mathcal{K} = (\mathcal{O}, \mathcal{P})$, the *ground instantiation of* $\mathcal{K}$ is the knowledge base $\mathcal{K}_G = (\mathcal{O}, \mathcal{P}_G)$ where $\mathcal{P}_G$ is obtained by replacing in each rule of $\mathcal{P}$ all variables with constants from $\mathcal{K}$ in all possible ways. Then it was shown in [7], that for a DL-safe hybrid knowledge base $\mathcal{K}$ and a ground MKNF formula ψ that $\mathcal{K} \models \psi$ if and only if $\mathcal{K}_G \models \psi$.

3 Well-founded MKNF Model

3.1 Three-valued models

Satisfiability as defined in [7] allows modal atoms only to be either true or false in a given MKNF structure. We extend the framework by allowing a third truth value **u**, denoting undefined, to be assigned to modal atoms while first-order atoms remain two-valued due to being interpreted solely in one first-order interpretation. We therefore evaluate MKNF sentences in consistent MKNF structures with respect to the set $\{\mathbf{t}, \mathbf{u}, \mathbf{f}\}$ of truth values with the order $\mathbf{f} < \mathbf{u} < \mathbf{t}$:

- $(I, M, N)(p(t_1, \ldots, t_n)) = \begin{cases} \mathbf{t} & \text{iff } p(t_1, \ldots, t_n) \in I \\ \mathbf{f} & \text{iff } p(t_1, \ldots, t_n) \notin I \end{cases}$
- $(I, M, N)(\neg\varphi) = \neg((I, M, N)(\varphi))$
- $(I, M, N)(\varphi_1 \wedge \varphi_2) = (I, M, N)(\varphi_1) \wedge (I, M, N)(\varphi_2)$
- $(I, M, N)(\varphi_1 \supset \varphi_2) = (I, M, N)(\varphi_1) \rightarrow (I, M, N)(\varphi_2)$
- $(I, M, N)(\exists x : \varphi) = \max\{(I, M, N)(\varphi[\alpha/x]) \mid \alpha \in \triangle\}$
- $(I, M, N)(\mathbf{K}\, \varphi) = \begin{cases} \mathbf{t} & \text{iff } (J, M, N)(\varphi) = \mathbf{t} \text{ for all } J \in M \\ \mathbf{f} & \text{iff } (J, M, N)(\varphi) = \mathbf{f} \text{ for some } J \in N \\ \mathbf{u} & \text{otherwise} \end{cases}$
- $(I, M, N)(\mathbf{not}\, \varphi) = \begin{cases} \mathbf{t} & \text{iff } (J, M, N)(\varphi) = \mathbf{f} \text{ for some } J \in N \\ \mathbf{f} & \text{iff } (J, M, N)(\varphi) = \mathbf{t} \text{ for all } J \in M \\ \mathbf{u} & \text{otherwise} \end{cases}$

where a *consistent* MKNF structure (I, M, N) does not allow any φ to be true for all $J \in M$ and false for some $J \in N$ at the same time. In case of max, the greatest element with respect to the truth ordering given above is chosen and, likewise, $\wedge$ is evaluated applying min wrt. the ordering. We evaluate $\neg$ as usual and $\rightarrow$ to true if the consequent has a truth value higher or equal than the antecedent and to false otherwise.

Note that an implication can never be undefined. The same holds for any objective MKNF formula and we thus obtain that formulas inside the DL base part of a hybrid MKNF knowledge base can never be undefined, so that evaluation in this case reduces to classical two-valued logic.

Definition 3.1. *An* interpretation pair (M, N) *consists of two MKNF interpretations* M, N *and* models *a closed MKNF formula* φ, *written* $(M, N) \models \varphi$, *if and only if* $(I, M, N)(\varphi) = \mathbf{t}$ *for each* $I \in M$. *We call* φ consistent *if there exists an interpretation pair modeling it.*

It is straightforward to see (cf. [7]) that (M, M) corresponds to the (two-valued) *MKNF interpretation* M, a nonempty set of Herbrand first-order interpretations over $\triangle$, since there are no undefined modal atoms in it. In this case, recalling from [8], M is additionally an *MKNF model* if (1) $(I, M, M)(\varphi) = \mathbf{t}$ for all $I \in M$ and (2) for each MKNF interpretation M' such that $M' \supset M$ we have $(I', M', M)(\varphi) = \mathbf{f}$ for some $I' \in M'$.

3.2 Alternating fixpoint for hybrid MKNF

As discussed in [7], since an MKNF model M is in general infinite, instead of representing M directly, a first-order formula φ is computed such that M is exactly the set of first-order models of φ. This is possible for modally closed MKNF formulae and the ideas from [9] are applied to provide a partition (P, N) of modal atoms which uniquely defines φ. We extend this idea by allowing partitions to be partial in the sense that modal atoms may occur neither in P nor in N, i.e. are neither true nor false but supposed to be undefined. To obtain the unique desired partial partition we apply a technique known from logic programming: stable models ([4]) for normal logic programs correspond one-to-one to MKNF models of programs of MKNF rules (see [6]). The well-founded model ([12]) for normal logic programs can be computed by an alternating fixpoint of the operator used to define stable models ([11]).

Here we proceed similarly: we define an operator which provides a stable condition for non-disjunctive hybrid MKNF knowledge bases and use it to obtain an alternating fixpoint which provides the well-founded MKNF model. We thus start by recalling some notions from [8] used to formalize partitions and the concepts related to them.

Definition 3.2. *Let* $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ *be a hybrid MKNF knowledge base. The* set of $\mathbf{K}$-atoms of $\mathcal{K}$, *written* $\mathsf{KA}(\mathcal{K})$, *is the smallest set that contains (i) all* $\mathbf{K}$*-atoms of* $\mathcal{P}_G$, *and (ii) a modal atom* $\mathbf{K}\,\xi$ *for each modal atom* $\mathbf{not}\,\xi$ *occurring in* $\mathcal{P}_G$.

For a subset P *of* $\mathsf{KA}(\mathcal{K})$, *the* objective knowledge *of* P *is the formula* $\mathsf{ob}_{\mathcal{K},P} = \mathcal{O} \cup \bigcup_{\mathbf{K}\,\xi \in P} \xi$. *A* (partial) partition (P, N) *of* $\mathsf{KA}(\mathcal{K})$ *is* consistent *if* $\mathsf{ob}_{\mathcal{K},P} \not\models \xi$ *for each* $\mathbf{K}\,\xi \in N$.

For a set of modal atoms S, S_{DL} *is the subset of DL-atoms of* S, $\widehat{S} = \{\xi \mid \mathbf{K}\,\xi \in S\}$, *and* $\widehat{S}_{DL} = \widehat{S'}$ *for* $S' = S_{DL}$.

An MKNF interpretation M induces *the partition* (P, N) *of* $\mathsf{KA}(\mathcal{K})$ *if* $\mathbf{K}\,\xi \in P$ *implies* $(M, M) \models \mathbf{K}\,\xi$ *and* $\mathbf{K}\,\xi \in N$ *implies* $(M, M) \models \mathbf{not}\,\xi$.

For the definition of the semantics we adapt the operators from [8]. These allow to draw conclusions from positive hybrid MKNF knowledge bases similarly to the immediate consequence operator for definite logic programs, only that the operators below also are “aware” of possible consequences drawn from the DL knowledge base $\mathcal{O}$.

Definition 3.3. *For $\mathcal{K}$ a positive non-disjunctive DL-safe hybrid MKNF knowledge base, $R_{\mathcal{K}}$, $D_{\mathcal{K}}$, and $T_{\mathcal{K}}$ are defined on the subsets of $\mathsf{KA}(\mathcal{K})$ as follows:*

$R_{\mathcal{K}}(S) = S \cup \{\mathbf{K}\, H \mid \mathcal{K}$ *contains a rule of the form (1) such that* $\mathbf{K}\, A_i \in S$ *for each* $1 \leq i \leq n\}$
$D_{\mathcal{K}}(S) = \{\mathbf{K}\, \xi \mid \mathbf{K}\, \xi \in \mathsf{KA}(\mathcal{K})$ *and* $\mathcal{O} \cup \widehat{S}_{DL} \models \xi\} \cup \{\mathbf{K}\, Q(b_1, \ldots, b_n) \mid \mathbf{K}\, Q(a_1, \ldots, a_n) \in S \setminus S_{DL}, \mathbf{K}\, Q(b_1, \ldots, b_n) \in \mathsf{KA}(\mathcal{K})$*, and* $\mathcal{O} \cup \widehat{S}_{DL} \models a_i \approx b_i$ *for* $1 \leq i \leq n\}$
$T_{\mathcal{K}}(S) = R_{\mathcal{K}}(S) \cup D_{\mathcal{K}}(S)$

The difference to the operators in [8] is that $D_{\mathcal{K}}$ cannot derive modal atoms not contained in $\mathsf{KA}(\mathcal{K})$.

As in [7], it can be shown that $T_{\mathcal{K}}$ is monotonic and yields a least fixpoint $T_{\mathcal{K}} \uparrow \omega$ in the usual manner. We can therefore, in the style of stable models, define a transformation which turns a non-disjunctive hybrid MKNF knowledge base into a positive one allowing to apply the previous operators, and an operator with the finxpoint of $T_{\mathcal{K}}$ on the transformed knowledge base.

Definition 3.4. *Let $\mathcal{K}_G = (\mathcal{O}, \mathcal{P}_G)$ be a ground non-disjunctive DL-safe hybrid MKNF knowledge base and $S \subseteq \mathsf{KA}(\mathcal{K})$. The* MKNF transform *$\mathcal{K}_G/S = (\mathcal{O}, \mathcal{P}_G/S)$ is obtained by $\mathcal{P}_G/S$ containing all rules $\mathbf{K}\, H \leftarrow \mathbf{K}\, A_1, \ldots, \mathbf{K}\, A_n$ for which there exists a rule $\mathbf{K}\, H \leftarrow \mathbf{K}\, A_1, \ldots, \mathbf{K}\, A_n, \mathbf{not}\, B_1, \ldots, \mathbf{not}\, B_m$ in $\mathcal{P}_G$ with $\mathbf{K}\, B_j \notin S$ for all $1 \leq j \leq m$.*

Definition 3.5. *Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a non-disjunctive DL-safe hybrid MKNF knowledge base and $S \subseteq \mathsf{KA}(\mathcal{K})$. We define:*

$$\Gamma_{\mathcal{K}}(S) = T_{\mathcal{K}_G/S} \uparrow \omega$$

This operator is antitonic (cf. extended technical report), and so applying $\Gamma_{\mathcal{K}}(S)$ twice is a monotonic operation yielding a least fixpoint by the Knaster-Tarski theorem (and dually a greatest one) and we can iterate as follows: $\Gamma^2_{\mathcal{K}} \uparrow 0 = \emptyset$, $\Gamma^2_{\mathcal{K}} \uparrow (n+1) = \Gamma^2_{\mathcal{K}}(\Gamma^2_{\mathcal{K}} \uparrow n)$, and $\Gamma^2_{\mathcal{K}} \uparrow \omega = \bigcup \Gamma^2_{\mathcal{K}} \uparrow i$, and dually $\Gamma^2_{\mathcal{K}} \downarrow 0 = \mathsf{KA}(\mathcal{K})$, $\Gamma^2_{\mathcal{K}} \downarrow (n+1) = \Gamma^2_{\mathcal{K}}(\Gamma^2_{\mathcal{K}} \downarrow n)$, and $\Gamma^2_{\mathcal{K}} \downarrow \omega = \bigcap \Gamma^2_{\mathcal{K}} \downarrow i$. The least and the greatest fixpoint then define the well-founded MKNF model.

Definition 3.6. *Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a non-disjunctive DL-safe hybrid MKNF knowledge base and let $\mathbf{P}_{\mathcal{K}}, \mathbf{N}_{\mathcal{K}} \subseteq \mathsf{KA}(\mathcal{K})$ with $\mathbf{P}_{\mathcal{K}} = \Gamma^2_{\mathcal{K}} \uparrow \omega$ and $\mathbf{N}_{\mathcal{K}} = \Gamma^2_{\mathcal{K}} \downarrow \omega$. Then the partial partition $(\mathbf{P}_{\mathcal{K}} \cup \{\mathbf{K}\, \pi(\mathcal{O})\}, \mathsf{KA}(\mathcal{K}) \setminus \mathbf{N}_{\mathcal{K}})$ is the* well-founded MKNF model *$W_{\mathcal{K}}$ of $\mathcal{K}$.*

Example 3.1. To illustrate the definitions, consider the following $\mathcal{K}$:

$p \sqsubseteq q$	$\mathbf{K}\, p \leftarrow \mathbf{not}\, q$	$\mathbf{K}\, S \leftarrow \mathbf{K}\, S$
$a \approx b$	$\mathbf{K}\, R \leftarrow \mathbf{not}\, R$	$\mathbf{K}\, Q(a) \leftarrow$

where we follow the convention of [7] that any predicate with capital letter is non-DL. We start with $\Gamma^2_{\mathcal{K}} \uparrow 0 = \emptyset$. Then $\Gamma_{\mathcal{K}}(\emptyset) = \{\mathbf{K}\,p, \mathbf{K}\,Q(a), \mathbf{K}\,R, \mathbf{K}\,q\}$ and $\Gamma_{\mathcal{K}}(\{\mathbf{K}\,p, \mathbf{K}\,Q(a), \mathbf{K}\,R, \mathbf{K}\,q\}) = \{\mathbf{K}\,Q(a)\} = \Gamma^2_{\mathcal{K}} \uparrow 1$ which is the least fixpoint. Likewise, we obtain the greatest fixpoint $\Gamma^2_{\mathcal{K}} \downarrow 1 = \{\mathbf{K}\,p, \mathbf{K}\,R, \mathbf{K}\,Q(a), \mathbf{K}\,q\}$ and the well-founded MKNF model is $W_{\mathcal{K}} = (\{\mathbf{K}\,Q(a), \mathbf{K}\,((p \subset q) \wedge (a \approx b))\}, \{\mathbf{K}\,S\})$. Note that $\mathbf{K}\,Q(b)$ is not contained in $W_{\mathcal{K}}$ since it does not occur in $\mathsf{KA}(\mathcal{K})$ even though it is of course a consequence of $\mathcal{K}$.

Now consider alternatively $\mathcal{K}'$ substituting the statement $p \sqsubseteq q$ by q. In this case we obtain the well-founded MKNF model $(\{\mathbf{K}\,Q(a), \mathbf{K}\,q, \mathbf{K}\,((p \subset q) \wedge (a \approx b))\}, \{\mathbf{K}\,S, \mathbf{K}\,p\})$. Note that both, $\mathcal{K}$ and $\mathcal{K}'$ do not have a single MKNF model.

Corollary 3.1. *Let $\mathcal{K} = (\mathcal{O}, \mathcal{P})$ be a non-disjunctive DL-safe hybrid MKNF KB and let $W_{\mathcal{K}}$ be the well founded model of of $\mathcal{K}$. Then $W_{\mathcal{K}}$ is consistent.*

Similarly to stable models, we can compute one fixpoint defining the well-founded MKNF model directly from the other.

Proposition 3.1. *Let $\mathcal{K}$ be a non-disjunctive DL-safe hybrid MKNF knowledge base. Then $\mathbf{P}_{\mathcal{K}} = \Gamma_{\mathcal{K}}(\mathbf{N}_{\mathcal{K}})$ and $\mathbf{N}_{\mathcal{K}} = \Gamma_{\mathcal{K}}(\mathbf{P}_{\mathcal{K}})$.*

Moreover, with this we can use $\Gamma_{\mathcal{K}}$ as an alternative characterization of MKNF models in case the considered knowledge base is consistent. It should be noted that in case of an inconsistent hybrid MKNF KB due to the operator $\mathcal{D}_{\mathcal{K}}$ we obtain a well-founded MKNF result where all modal **K**-atoms are true (though not necessarily a model of $\mathcal{K}$). I.e., even though we always obtain a well-founded MKNF model for any $\mathcal{K}$ the result may not be a model.

It was also shown that the information derived in the well-founded MKNF model is contained in any MKNF model, and that the well-founded MKNF model is a model in the three-valued framework we defined in the previous subsection.

Theorem 3.1. *Let $\mathcal{K}$ be a non-disjunctive DL-safe hybrid MKNF knowledge base, M an MKNF model of $\mathcal{K}$ with (P, N) induced by M, and $W_{\mathcal{K}} = (P_W, N_W)$ the well-founded model of $\mathcal{K}$. Then $P_W \subseteq (P \cup \{\mathbf{K}\,\pi(\mathcal{O})\})$ and $N_W \subseteq N$.*

Theorem 3.2. *Let $\mathcal{K}$ be a consistent non-disjunctive DL-safe hybrid MKNF KB and $(\mathbf{P}_{\mathcal{K}} \cup \{\mathbf{K}\,\pi(\mathcal{O})\}, \mathsf{KA}(\mathcal{K}) \setminus \mathbf{N}_{\mathcal{K}})$ be the well-founded model of $\mathcal{K}$. Then $(I_P, I_N) \models \pi(\mathcal{K})$ where $I_P = \{I \mid I \models \mathsf{ob}_{\mathcal{K},\mathbf{P}_{\mathcal{K}}}\}$ and $I_N = \{I \mid I \models \mathsf{ob}_{\mathcal{K},\mathbf{N}_{\mathcal{K}}}\}$.*

One of the open questions mentioned in [8] was that MKNF models are not compatible with the well-founded model for logic programs. Our approach, regarding knowledge bases just consisting of rules, does coincide with the well-founded model for the corresponding (normal) logic program.

Finally, though not providing here a detailed study of complexity issues we can recall from [7] that assuming that entailment of first-order formulas encountered while computing $T_{\mathcal{K}}$ is decidable in $\mathcal{C}$ the complexity of computing $T_{\mathcal{K}}$ is in $\mathsf{P}^{\mathcal{C}}$ (for positive nondisjunctive programs). Since we just apply the same operator n-times we should remain in the same complexity class while the complexity for reasoning with MKNF models in nondisjunctive programs is shown to be $\mathcal{E}^{\mathsf{P}^{\mathcal{C}}}$ where $\mathcal{E} = \mathsf{NP}$ if $\mathcal{E} \subseteq \mathsf{NP}$, and $\mathcal{E} = \mathcal{C}$ otherwise. Thus computing the well-founded MKNF model ends up in a strictly smaller complexity class than deriving the MKNF models.

4 Conclusions and Future Work

We have continued the work on hybrid MKNF knowledge bases providing an alternating fixpoint restricted to nondisjunctive rules. We basically achieve better complexity results by having only one model which is semantically weaker than any MKNF model defined in [8] and bottom-up computable. The well-founded MKNF model is not only a sound approximation of any MKNF model but a partition of modal atoms which can seemlessly be integrated in the reasoning algorithms presented for MKNF models in [8] thus reducing the difficulty of guessing the 'right' model. Future work shall include the extension to disjunctive rules, a study on top-down querying procedures, and further investigations on the well-founded MKNF model in the three-valued framework.

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